3-6 Applied Calculus Solutions

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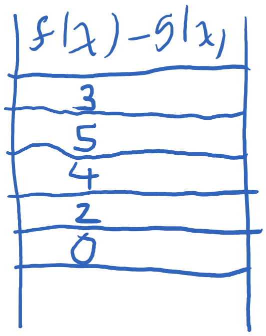
In problems 1 – 4, use the values in the table to estimate the areas.

x	$f(x)$	$g(x)$	$h(x)$
0	5	2	5
1	6	1	6
2	6	2	8
3	4	2	6
4	3	3	5
5	2	4	4
6	2	0	2



1. Estimate the area between f and g, between $x = 0$ and $x = 4$.

We calculate $f(x) - g(x)$ for $x = 0,1,2,3,4$.

We calculate the area of rectangles, each with base 1 and height $f(x) - g(x)$.
Using left endpoints, our estimate is : $3 + 5 + 4 + 2 = \boxed{14}$

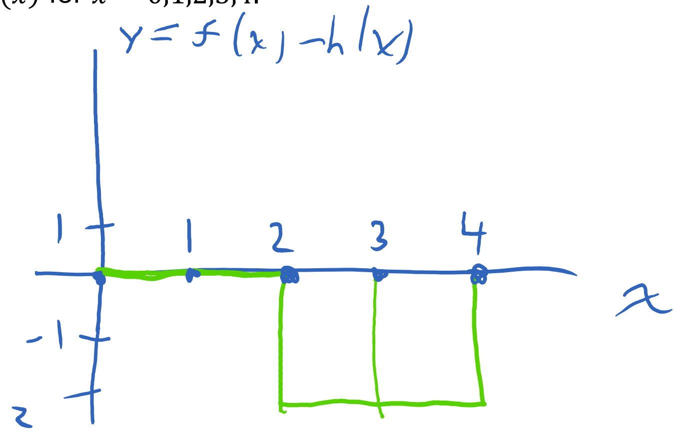
Using right endpoints, our estimate is :$5 + 4 + 2 + 0 = \boxed{11}$

Taking the average, our estimate is $\frac{14+11}{2} = \boxed{\frac{25}{2} = 12.5}$

3. Estimate the area between f and h, between $x = 0$ and $x = 4$.

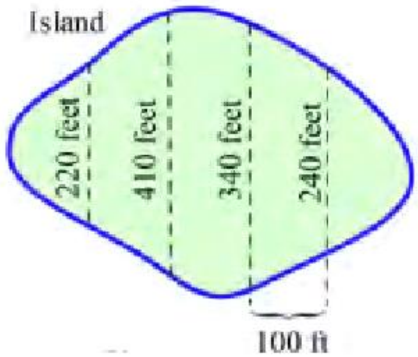
We calculate $f(x) - h(x)$ for $x = 0,1,2,3,4$.

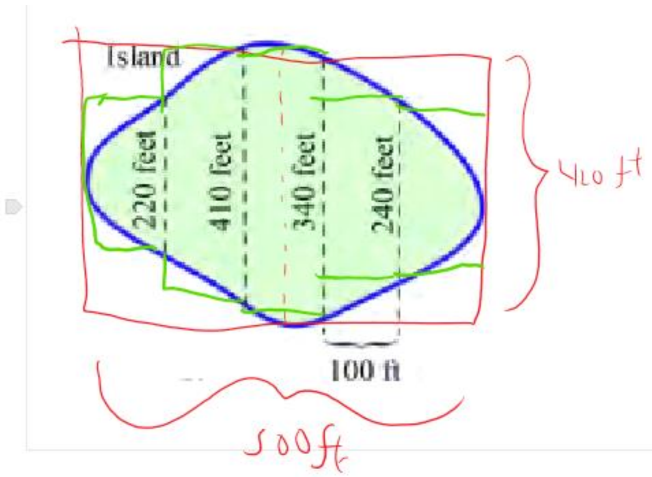
x	f(x) - h(x)
0	0
1	0
2	-2
3	-2
4	-2



For the first two rectangles, the area is 0.
For the next two rectangles, the area is -4.
Therefore, we estimate that the area between the two curves is 4.

5. Estimate the area of the island shown



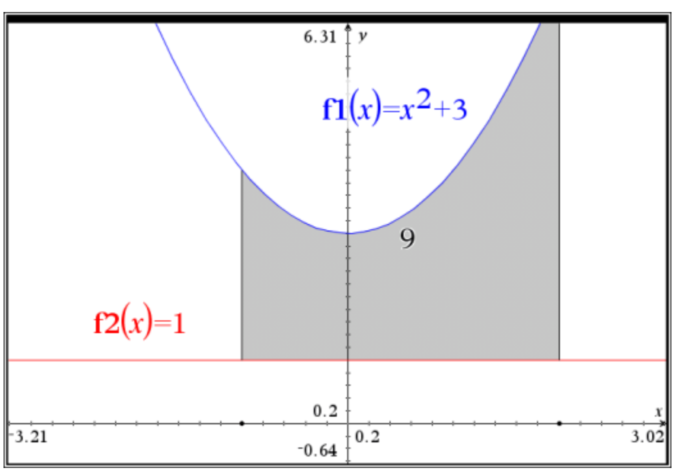


Using the large red rectangle, we estimate is 500*420=210,000 square feet - overestimate

Using the green rectangles, we estimate 100*(220+410+340+240+240)=145,000 square feet

In problems 6 – 15, find the **area** between the graphs of f and g for x in the given interval. Remember to draw the graph!

6. $f(x) = x^2 + 3$, $g(x) = 1$ and $-1 \le x \le 2$.

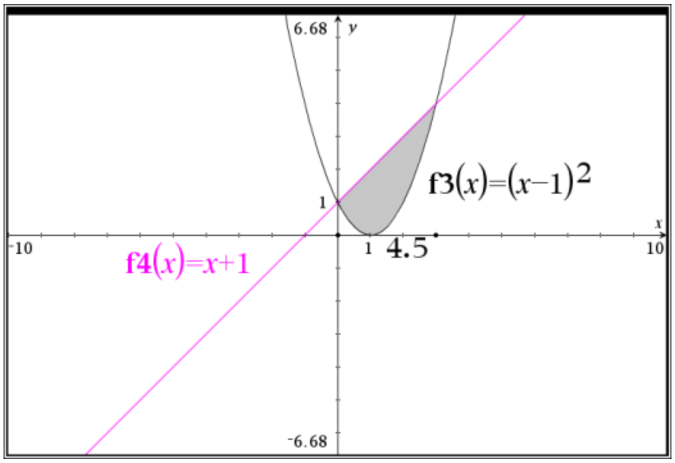


$$\text{area} = \int_{-1}^{2} [(x^2+1)-1]dx = \int_{-1}^{2} x^2 dx$$

$$= \left[\frac{x^3}{3}\right]_{-1}^{2} = \left(\frac{2^3}{3}\right) - \left(\frac{(-1)^3}{3}\right)$$

$$= \left(\frac{8}{3}\right) - \left(\frac{-1}{3}\right) = \boxed{3}$$

9. $f(x) = (x-1)^2$, $g(x) = x+1$ and $0 \le x \le 3$.



In order to determine the limits of integraton and which function to subtract, we find the intersection points of the two curves.

$(x-1)^2 = x+1$
$\Rightarrow x^2 - 2x + 1 = x + 1$
$\Rightarrow x^2 - 2x + 1 - x - 1 = 0$
$\Rightarrow x^2 - 3x = 0$
$\Rightarrow x(x-3) = 0$
$\Rightarrow x = 0, 3$, which are the given left and right bounds of x.
To see which curve is above the other, we need only check any
one point in the interval $(0,3)$.
Let $x = 1$.
$f(1) = (1-1)^2 = 0$
$g(1) = 1 + 1 = 2$
$2 > 0 \Rightarrow g(x) \ge f(x)$ on the interval $[0,3]$

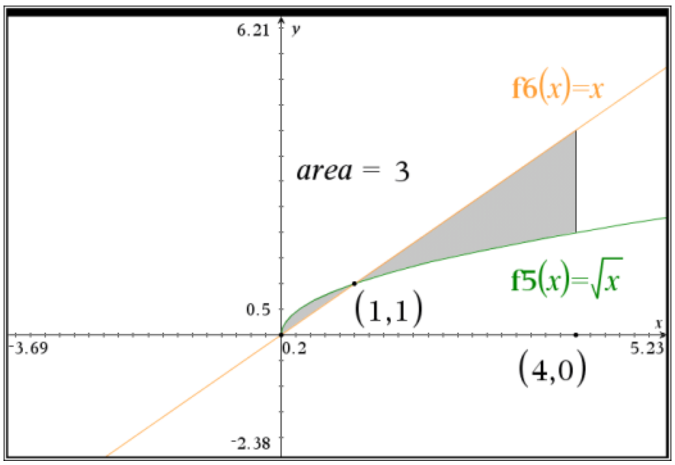
$$\text{area} = \int_0^3 [(x+1)-(x-1)^2]dx = \int_0^3 (3x - x^2)dx$$

$$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = \left(\frac{3(3)^2}{2} - \frac{3^3}{3}\right) - \left(\frac{3(0)^2}{2} - \frac{0^3}{3}\right)$$

$$= \left(\frac{27}{2} - \frac{27}{3}\right) - 0 = \boxed{\frac{9}{2}}$$

11. $f(x) = \sqrt{x}$, $g(x) = x$ and $0 \le x \le 4$.

TI-nspire allows us to graph the bounded area and calculate it



In order to determine the limits of integraton and which function to subtract, we find the intersection points of the two curves.

$\sqrt{x} = x$

$\Rightarrow x = x^2$

$\Rightarrow x^2 - x = 0$

$\Rightarrow x(x-1) = 0$

$\Rightarrow x = 0, 1$

To see which curve is above the other, we need only check any one point in the interval $(0, 1)$ and any one point in the interval $(1, 4)$.

Let $x = \frac{1}{2}$.

$f\left(\frac{1}{2}\right) = \sqrt{\frac{1}{2}} \approx 0.70711$

$g\left(\frac{1}{2}\right) = \frac{1}{2} = 0.5$

$0.70711 > 0.5 \Rightarrow f(x) \geq g(x)$ on the interval $[0, 1]$.

Let $x = 2$.

$f(2) = \sqrt{2} \approx 1.4142$

$g(2) = 2$

$2 > 1.4142 \Rightarrow g(x) \geq f(x)$ on the interval $[1, 4]$.

$$\text{area} = \int_0^1 (\sqrt{x} - x)dx + \int_1^4 (x - \sqrt{x})dx$$

$$= \int_0^1 (x^{1/2} - x)dx + \int_1^4 (x - x^{1/2})dx$$

$$= \left[\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - \frac{2x^{3/2}}{3}\right]_1^4$$

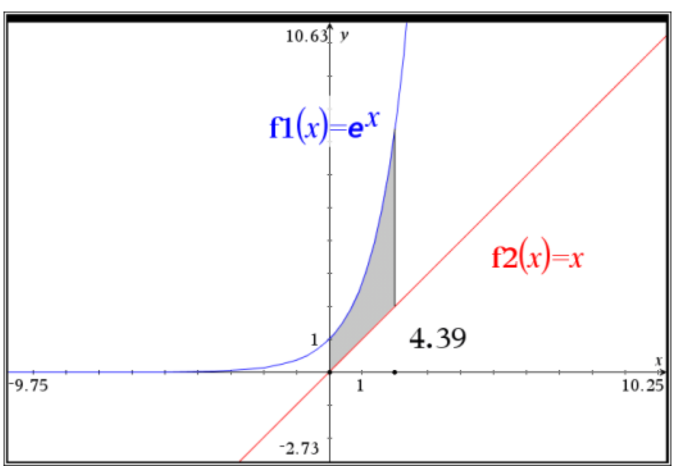
$$= \left(\frac{2(1)^{3/2}}{3} - \frac{(1)^2}{2}\right) - \left(\frac{2(0)^{3/2}}{3} - \frac{(0)^2}{2}\right) + \left(\frac{(4)^2}{2} - \frac{2(4)^{3/2}}{3}\right) - \left(\frac{(1)^2}{2} - \frac{2(1)^{3/2}}{3}\right)$$

$$= \left(\frac{2}{3} - \frac{1}{2}\right) - \left(\frac{0}{3} - \frac{0}{2}\right) + \left(\frac{16}{2} - \frac{16}{3}\right) - \left(\frac{1}{2} - \frac{2}{3}\right)$$

$$= \left(\frac{2}{3} - \frac{16}{3} + \frac{2}{3}\right) + \left(-\frac{1}{2} + \frac{16}{2} - \frac{1}{2}\right)$$

$$= (-4) + (7) = \boxed{3}$$

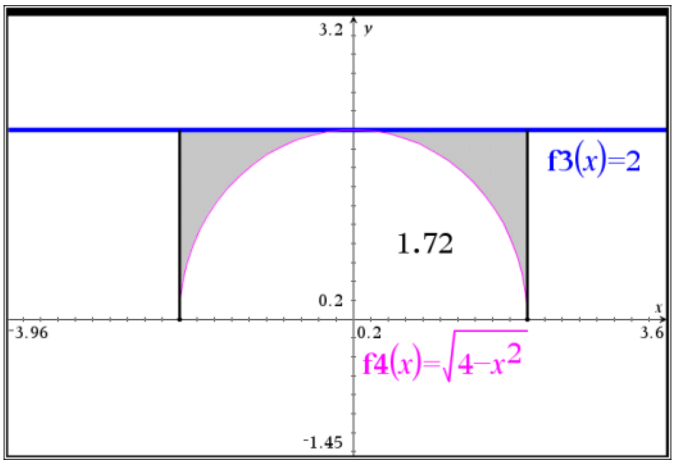
13. $f(x) = e^x$, $g(x) = x$ and $0 \leq x \leq 2$.



Area = $\int_0^2 (e^x - x)dx = \left[e^x - \frac{x^2}{2}\right]_0^2 = \left(e^2 - \frac{2^2}{2}\right) - \left(e^0 - \frac{0^2}{2}\right)$

$= (e^2 - 2) - (1 - 0) = \boxed{e^2 - 3 \approx 4.3891}$

15. $f(x) = 2$, $g(x) = \sqrt{4 - x^2}$ and $-2 \le x \le 2$.



Area = $\int_{-2}^{2} \left(2 - \sqrt{4 - x^2}\right)dx = \int_{-2}^{2} 2dx - \int_{0-2}^{2} \sqrt{4 - x^2}\,dx$

In this course, we do not cover the trigonometric substitution which would allow us to evaluate the second integral. However, we can evaluate each integral with elementary geometry.

$\int_{-2}^{2} 2dx$ = area of a rectangle with base 4 and height 2 = $(4)(2) = \boxed{8}$

$\int_{-2}^{2} \sqrt{4 - x^2}\,dx$ = area of semicircle, with radius 2 = $\frac{\pi(2^2)}{2} = \boxed{2\pi}$

$\therefore$ $\boxed{\text{area} = 8 - 2\pi \approx 1.7168}$

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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1*
by Shana Calaway, Dale Hoffman, David Lippman

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