3-4 Applied Calculus Solutions

Saturday, July 2, 2016 5:25 PM

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For problems 1-8, find the indicated antiderivative.

1. $\int \frac{1}{(4x+1)^3}\,dx$

Let $I = \int \frac{1}{(4x+1)^3}\,dx$

Let $u = 4x + 1$

Then $du = 4dx \Rightarrow dx = \frac{du}{4}$

$\Rightarrow I = \int \frac{1}{u^3}\frac{du}{4} = \frac{1}{4}\int u^{-3}du$

$= \frac{1}{4}\left[\frac{u^{-2}}{-2}\right] = -\frac{1}{8}\left(\frac{1}{u^2}\right) = \boxed{-\frac{1}{8(4x+1)^2} + C}$

3. $\int (1.0003)^{12t}\,dt$

Let $I = \int (1.0003)^{12t} dt = \frac{277.82}{10000.0^{12.0t}} 10003.^{12.0t} = \frac{277.82}{10000.0^{12.0t}} 10003.^{12.0t}$

Let $u = 12t$

$\Rightarrow du = 12dt$

$\Rightarrow dt = \frac{du}{12}$

$\Rightarrow I = \frac{1}{12} \int (1.0003)^u du$

$= \frac{1}{12} \frac{(1.0003)^u}{\ln 1.0003} + C$

$= \boxed{\frac{1}{12} \frac{(1.0003)^{12t}}{\ln 1.0003} + C}$

5. $\int \sqrt{w+5}\, dw$

Let $I = \int \sqrt{w+5}\, dw$

Let $u = w + 5$

$\Rightarrow du = dw$

$\Rightarrow I = \int \sqrt{u}\, dw = \int u^{1/2} dw$

$= \frac{u^{3/2}}{3/2} + C = \frac{2u^{3/2}}{3} + C$

$\therefore \boxed{I = \frac{2(w+5)^{3/2}}{3} + C}$

7. $\int \frac{dx}{x \ln x}$

Let $I = \int \frac{dx}{x \ln x}$

Let $u = \ln x$

$\Rightarrow du = \frac{dx}{x}$

$\Rightarrow I = \int \frac{du}{u} = \ln|u| + C$

$\therefore \boxed{I = \ln|\ln x| + C}$

For problems 9-12, find an antiderivative of the integrand and use the Fundamental Theorem to evaluate the definite integral.

9. $\int_{-2}^{2} \frac{2x}{1+x^2}\,dx$

Let $I = \int_{-2}^{2} \frac{2x}{1+x^2}\,dx$

Let $u = 1 + x^2$

$\Rightarrow du = 2x dx$

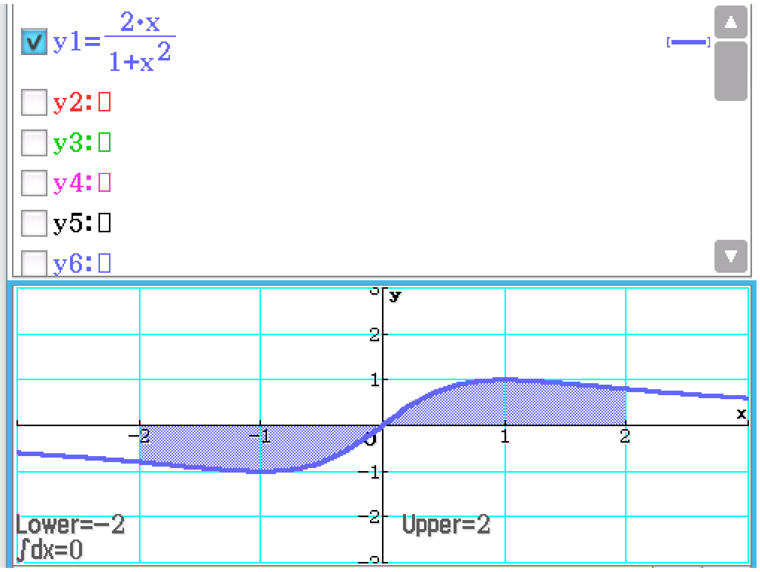
$x = -2 \Rightarrow u = 1 + (-2)^2 = 5$

$x = 2 \Rightarrow u = 1 + (2)^2 = 5$

$\Rightarrow I = \int_5^5 \frac{du}{u} = \ln|5| - \ln|5| = \boxed{0}$

However, we can save some work
by noting that $\int_a^a f(x)dx = 0$ for any function $f(x)$.

Also, because $\frac{2x}{1+x^2}$ is an odd function, and the interval of integration is symmetric with respect to
the $y-axis$, the areas
to the left and right of the $y-axis$ will cancel each other



11. $\int_{2}^{4} (x-2)^3 \, dx$

Let $I = \int_2^4 (x-2)^3 dx$

Let $u = x - 2$

$\Rightarrow du = dx$

$x = 2 \Rightarrow u = 2 - 2 = 0$

$x = 4 \Rightarrow u = 4 - 2 = 2$

$\Rightarrow I = \int_0^2 u^3 du = \left[\frac{u^4}{4} \right]_0^2 = \frac{2^4}{4} - \frac{0^4}{4} = \frac{16}{4} - 0$

$\therefore \boxed{I = 4}$

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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1*
by Shana Calaway, Dale Hoffman, David Lippman