3-2 Applied Calculus Solutions

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In problems 1 – 5, verify that $F(x)$ is an antiderivative of the integrand $f(x)$ and use Part 2 of the Fundamental Theorem to evaluate the definite integrals.

1. $\int_0^1 2x \, dx$, $F(x) = x^2 + 5$

$$\frac{dF}{dx} = \frac{d(x^2+5)}{dx} = 2x + 0 = 2x$$

$$\int_0^1 2x\,dx = [x^2+5]_0^1 = (1^2+5) - (0^2+5) = 1 + 5 - 0 - 5 = \boxed{1}$$

3. $\int_1^3 x^2 \, dx$, $F(x) = \frac{1}{3} x^3$

$$\frac{dF}{dx} = \frac{d\left(\frac{x^3}{3}\right)}{dx} = \frac{1}{3}(3)x^2 = x^2$$

$$\int_1^3 x^2\,dx = \left[\frac{x^3}{3}\right]_1^3 = \left(\frac{3^3}{3}\right) - \left(\frac{1^3}{3}\right) = 9 - \frac{1}{3} = \boxed{\frac{26}{3}}$$

5. $\int_1^5 \frac{1}{x} \, dx$, $F(x) = \ln(x)$

$$\frac{dF}{dx} = \frac{d(\ln(x))}{dx} = \frac{1}{x}$$

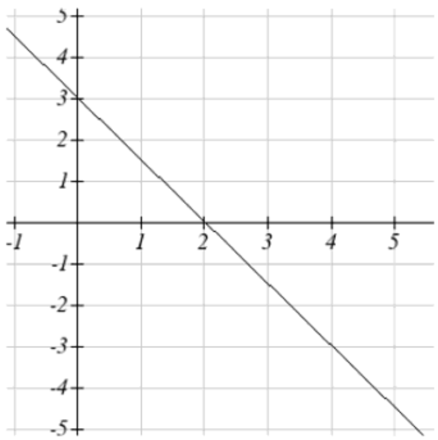
$$\int_1^5 \frac{1}{x}\,dx = [\ln(x)]_1^5 = \ln(5) - \ln(1) = \ln(5) - 0 = \boxed{\ln 5}$$

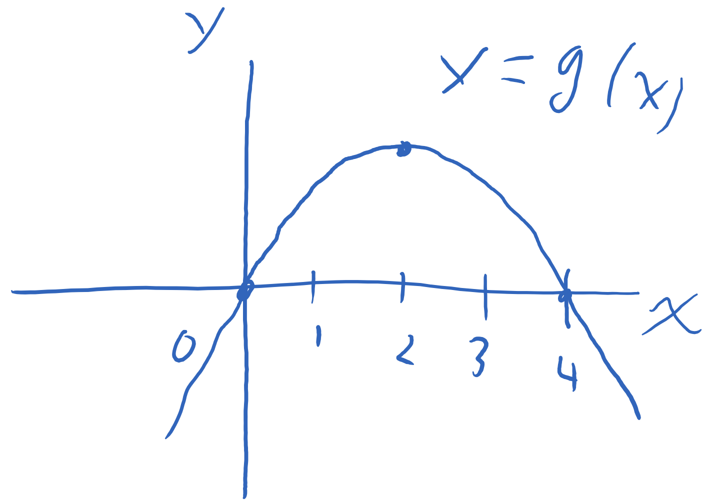
7. Given $A(x) = \int_0^x (3 - t^2)\ dt$, find A'(x)

$A'(x) = 3 - x^2$, by the Fundamental Theorem of Calculus (Part 2)

For problems 9-10, the graph provided shows $g'(x)$. Use it sketch a graph of $g(x)$ that satisfies $g(0) = 0$.

9.





We can use example 7 as a model.

Recall from the last chapter the relationships between the function graph and the derivative graph:

f(x)	increasing	Decreasing	Concave up	Concave down
f '(x)	+	-	Increasing	decreasing
f "(x)			+	-

From the graph, we see that $g'(x) > 0$ for $x \leq 2$ and $g'(x) \leq 0$ for $2 \leq x$.
Thus, g(x) is increasing for $x \leq 2$ and decreasing for $2 \leq x$.
$g'(2) = 0$, making $x = 2$ a critical point.
By the first derivative test, this is a local maximum.
This gives us the graph to the above right.

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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1* by Shana Calaway, Dale Hoffman, David Lippman