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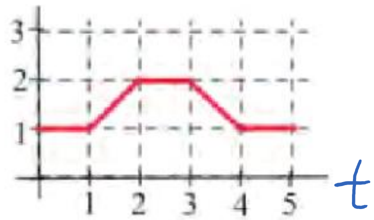
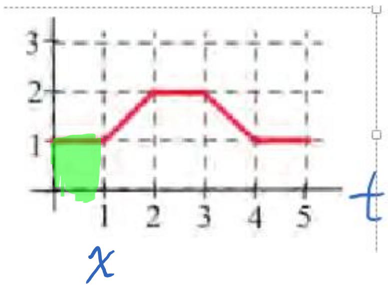
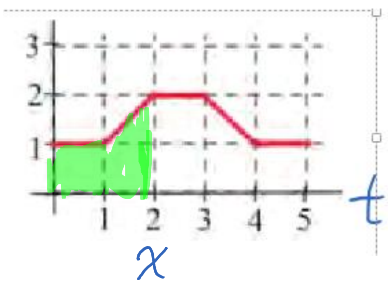
1. Let $A(x)$ represent the area bounded by the graph and the horizontal axis and vertical lines at $t=0$ and $t=x$ for the graph shown. Evaluate $A(x)$ for $x = 1, 2, 3, 4,$ and 5.



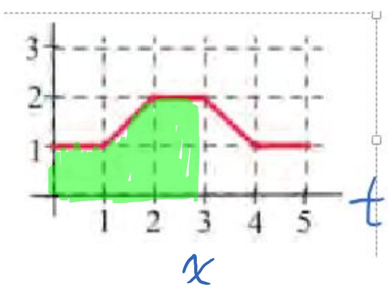


$$A(1) = (1)(1) = \boxed{1}$$

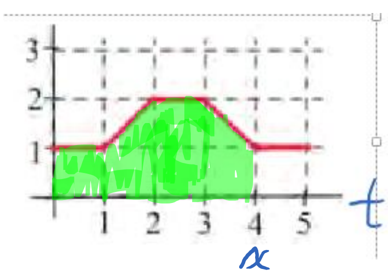


$$A(1) = (1)(1) = \boxed{1}$$

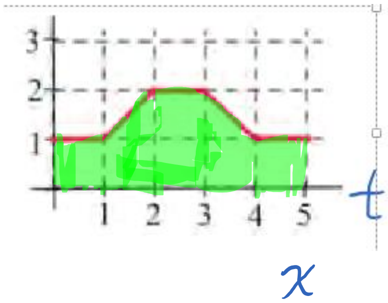
$$A(2) = A(1) + (1)(1) + \left(\frac{1}{2}\right)(1)(1) = 1 + (1)(1) + \left(\frac{1}{2}\right)(1)(1) = \boxed{\frac{5}{2}}$$



$$A(3) = A(2) + (1)(2) = \frac{5}{2} + 2 = \boxed{\frac{9}{2}}$$

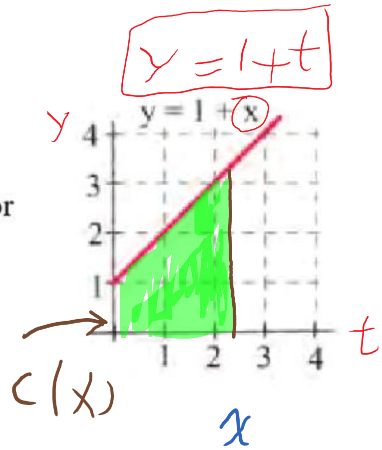


$$A(4) = A(3) + (1)(1) + \left(\frac{1}{2}\right)(1)(1) = \frac{9}{2} + (1)(1) + \left(\frac{1}{2}\right)(1)(1) = \boxed{6}$$



$A(5) = A(4) + (1)(1) = 6 + 1 = \boxed{7}$

3. Let C(x) represent the area bounded by the graph and the horizontal axis and vertical lines at t=0 and t=x for the graph shown. Evaluate C(x) for x = 1, 2, and 3 and find a formula for C(x).



$C(1) = 1 + \frac{1}{2} = \boxed{\frac{3}{2}}$

$C(2) = C(1) + (2)(1) + \frac{1}{2} = \frac{3}{2} + (2)(1) + \frac{1}{2} = \boxed{4}$

$C(3) = C(2) + (3)(1) + \frac{1}{2} = 4 + (3)(1) + \frac{1}{2} = \boxed{\frac{15}{2}}$

$C(x)$ = area of rectangle with base x and height 1

+ area of triangle with base x and height $1 + x - 1 = x$

$\Rightarrow C(x) = (x)(1) + \left(\frac{1}{2}\right)(x)(x)$

$\therefore \boxed{C(x) = x + \frac{x^2}{2}}$

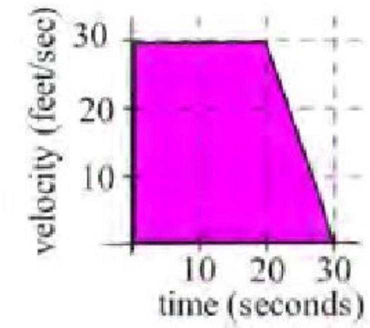
We can check this by calculating $C(1)$, $C(2)$, and $C(3)$.

$C(1) = 1 + \frac{1^2}{2} = \boxed{\frac{3}{2}}$

$C(2) = 2 + \frac{2^2}{2} = \boxed{4}$

$C(3) = 3 + \frac{3^2}{2} = \boxed{\frac{15}{2}}$

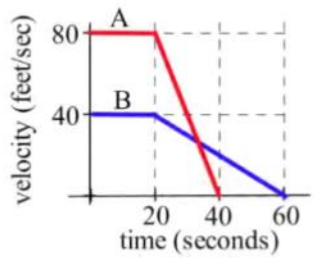
5. A car had the velocity shown in the graph to the right. How far did the car travel from t= 0 to t = 30 seconds?



The distance = the area of the rectangle with base 20 and height 30
+ the area of the triangle with base 10 and height 30

$$= (20\,\text{s})\left(30\,\frac{\text{ft}}{\text{s}}\right) + \left(\frac{1}{2}\right)(10\,\text{s})\left(30\,\frac{\text{ft}}{\text{s}}\right) = \boxed{750\,\text{ft}}$$

7. The velocities of two cars are shown in the graph.
(a) From the time the brakes were applied, how many seconds did it take each car to stop?
(b) From the time the brakes were applied, which car traveled farther until it came to a complete stop?



(7a)

Car A moved with a constant velocity of $80\,\frac{\text{ft}}{\text{s}}$ for 20 seconds.
Car A then took $40\,\text{s} - 20\,\text{s} = \boxed{20\,\text{s}}$ to stop.

Car B moved with a constant velocity of $40\,\frac{\text{ft}}{\text{s}}$ for 20 seconds.
Car B then took $60\,\text{s} - 20\,\text{s} = \boxed{40\,\text{s}}$ to stop.

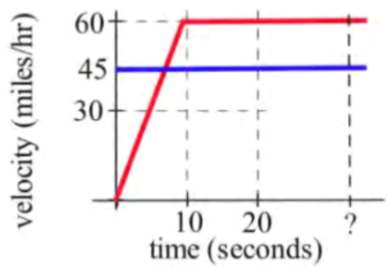
(7b)

From the time the brakes were applied, the distance traveled by car A
= area of triangle with base $20\,\text{s}$ and height $80\,\frac{\text{ft}}{\text{s}} = \left(\frac{1}{2}\right)(20\,\text{s})\left(80\,\frac{\text{ft}}{\text{s}}\right) = \boxed{800\,\text{ft}}$.

From the time the brakes were applied, the distance traveled by car B
= area of triangle with base $40\,\text{s}$ and height $40\,\frac{\text{ft}}{\text{s}} = \left(\frac{1}{2}\right)(40\,\text{s})\left(40\,\frac{\text{ft}}{\text{s}}\right) = \boxed{800\,\text{ft}}$.

$\therefore$ Both cars traveled the **same distance** to stop.

9. Police chase: A speeder traveling 45 miles per hour (in a 25 mph zone) passes a stopped police car which immediately takes off after the speeder. If the police car speeds up steadily to 60 miles/hour in 20 seconds and then travels at a steady 60 miles/hour, **how long** and **how far** before the police car catches the speeder who continued traveling at 45 miles/hour?



Let $P(t)$ = distance traveled by police car after time t, measured in seconds.
Let $S(t)$ = distance traveled by the speeder after tim t, measured in seconds.

We must find t such that $P(t) = S(t)$.

$P(t)$ = are of triangle with base $10\,\text{s}$ and height $60\,\frac{\text{mi}}{\text{h}}$

+ area of rectangle with base $t - 10\,\text{s}$ and height $60\,\frac{\text{mi}}{\text{h}}$

$$= \left(\frac{1}{2}\right)(10\,\text{s})\left(60\,\frac{\text{mi}}{\text{h}}\right) + (t - 10\,\text{s})\left(60\,\frac{\text{mi}}{\text{h}}\right)$$

$$= \left(\frac{1}{2}\right)(10\,\text{s})\left(60\,\frac{\text{mi}}{3600\,\text{s}}\right) + (t - 10\,\text{s})\left(60\,\frac{\text{mi}}{3600\,\text{s}}\right)$$

$$= \frac{1}{12}\,\text{mi} + (t - 10\,\text{s})\left(\frac{1}{60}\,\frac{\text{mi}}{\text{s}}\right) = \frac{1}{12}\,\text{mi} + \frac{t}{60}\,\frac{\text{mi}}{\text{s}} - \frac{10\,\text{s}}{60}\,\frac{\text{mi}}{\text{s}}$$

$$= \frac{1}{12}\,\text{mi} - \frac{1\,\text{mi}}{6} + \frac{t}{60}\,\frac{\text{mi}}{\text{s}}$$

$$\boxed{P(t) = -\frac{1}{12}\,\text{mi} + \frac{t}{60}\,\frac{\text{mi}}{\text{s}}}$$

$S(t)$ = the area of the rectangle with base t and height $45\,\frac{\text{mi}}{\text{h}} = 45\,\frac{\text{mi}}{3600\,\text{s}}$

$$\boxed{S(t) = \frac{t}{80}\,\frac{\text{mi}}{\text{s}}}$$

$$P(t) = S(t) \Rightarrow -\frac{1}{12}\,\text{mi} + \frac{t}{60}\,\frac{\text{mi}}{\text{s}} = \frac{t}{80}\,\frac{\text{mi}}{\text{s}}$$

$$\Rightarrow \frac{t}{60}\,\frac{\text{mi}}{\text{s}} - \frac{t}{80}\,\frac{\text{mi}}{\text{s}} = \frac{1}{12}\,\text{mi}$$

$$\Rightarrow \frac{t}{60\,\text{s}} - \frac{t}{80\,\text{s}} = \frac{1}{12}$$

$$\Rightarrow \frac{t}{60} - \frac{t}{80} = \frac{1\,\text{s}}{12}$$

$$\Rightarrow t\left(\frac{1}{60} - \frac{1}{80}\right) = \frac{1\,\text{s}}{12}$$

$$\Rightarrow t\left(\frac{1}{240}\right) = \frac{1\,\text{s}}{12}$$

$$\Rightarrow t = \frac{240\text{ s}}{12}$$

$$\therefore \boxed{t = 20\text{ s}}$$

We can use either distance formula to find out how far each cars goes until they meet.

$$S(20\text{ s}) = \frac{20\text{ s}}{80}\frac{\text{mi}}{\text{s}} = \boxed{\frac{1}{4}\text{ mi}}$$

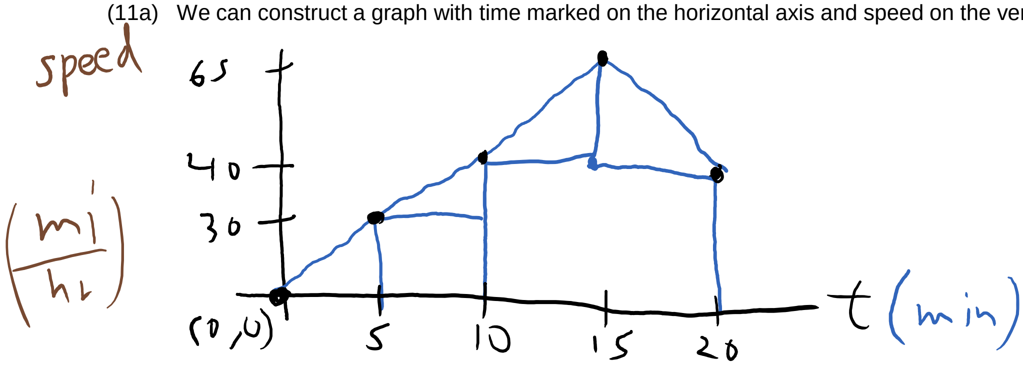
$$P(20\text{ s}) = -\frac{1}{12}\text{ mi} + \frac{20\text{ s}}{60}\frac{\text{mi}}{\text{s}} = \boxed{\frac{1}{4}\text{ mi}}$$

11. The table shows the speedometer readings for a short car trip.

t, in minutes	0	5	10	15	20
Speed, in mph	0	30	40	65	40

a. Use the table to estimate how far the car traveled over the twenty minutes shown.
b. How accurate would you expect your estimate to be?

(11a) We can construct a graph with time marked on the horizontal axis and speed on the vertical axis.



We can connect the plotted points with line segments and calculate the total area

by adding the areas of the triangles and rectangles.

$$\text{distance} \approx \left(\frac{1}{2}\right)(5\text{ min})\left(30\frac{\text{mi}}{\text{h}}\right) + \left(\frac{1}{2}\right)(5\text{ min})\left(10\frac{\text{mi}}{\text{h}}\right) + \left(\frac{1}{2}\right)(5\text{ min})\left(25\frac{\text{mi}}{\text{h}}\right) + \left(\frac{1}{2}\right)(5\text{ min})\left(25\frac{\text{mi}}{\text{h}}\right)$$
$$+(5\text{ min})\left(30\frac{\text{mi}}{\text{h}}\right) + (10\text{ min})\left(40\frac{\text{mi}}{\text{h}}\right)$$

$$= \left(\frac{1}{2}\right)(5\text{ min})\left(30\frac{\text{mi}}{60\text{ min}}\right) + \left(\frac{1}{2}\right)(5\text{ min})\left(10\frac{\text{mi}}{60\text{ min}}\right) + \left(\frac{1}{2}\right)(5\text{ min})\left(25\frac{\text{mi}}{60\text{ min}}\right) + \left(\frac{1}{2}\right)(5\text{ min})\left(25\frac{\text{mi}}{60\text{ min}}\right)$$
$$+(5\text{ min})\left(30\frac{\text{mi}}{60\text{ min}}\right) + (10\text{ min})\left(40\frac{\text{mi}}{60\text{ min}}\right)$$

$$= \left(\frac{1}{2}\right)(5)\left(30\frac{\text{mi}}{60}\right) + \left(\frac{1}{2}\right)(5)\left(10\frac{\text{mi}}{60}\right) + \left(\frac{1}{2}\right)(5)\left(25\frac{\text{mi}}{60}\right) + \left(\frac{1}{2}\right)(5)\left(25\frac{\text{mi}}{60}\right)$$
$$+(5)\left(30\frac{\text{mi}}{60}\right) + (10)\left(40\frac{\text{mi}}{60}\right)$$

$$= \left[\left(\frac{1}{2}\right)(5)\left(\frac{30}{60}\right) + \left(\frac{1}{2}\right)(5)\left(\frac{10}{60}\right) + \left(\frac{1}{2}\right)(5)\left(\frac{25}{60}\right) + \left(\frac{1}{2}\right)(5)\left(\frac{25}{60}\right) + (5)\left(\frac{30}{60}\right) + (10)\left(\frac{40}{60}\right)\right]\text{mi}$$

$$= \frac{155}{12}\text{ mi} \approx \boxed{12.9\text{ mi}}$$

(11b) We have no precise way of determining the accuracy of this estimate.
The speed could vary widely between the recorded points.

We could also follow the method of Example 9 and estimate the area with rectangles, using left endpoints and right endpoints, and averaging the results.

13. The table shows values of $g(x)$.

x	0	1	2	3	4	5	6
$g(x)$	140	142	144	152	154	165	200

Use the table to estimate

a. $\int_0^3 g(x)dx$ b. $\int_3^6 g(x)dx$ c. $\int_0^6 g(x)dx$

(13a) Each rectangle has base 1.

Using left endpoints,

$$\int_0^3 g(x)dx \approx 0 + 142 + 144 = \boxed{286}$$

Using right endpoints,

$$\int_0^3 g(x)dx \approx 142 + 144 + 152 = \boxed{438}$$

Taking the average, $\int_0^3 g(x)dx \approx \frac{286 + 438}{2} = \boxed{362}$

(13b)

Using left endpoints,

$$\int_3^6 g(x)dx \approx 152 + 154 + 165 = 471$$

Using right endpoints,

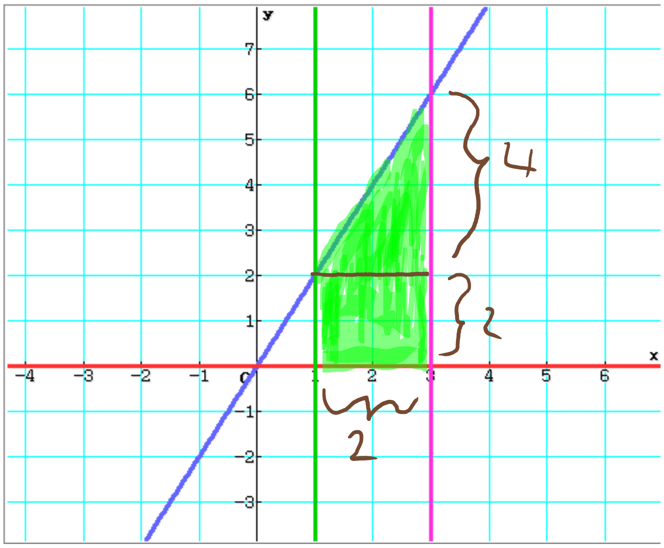
$$\int_3^6 g(x)dx \approx 154 + 165 + 200 = 519$$

Taking the average, $\int_3^6 g(x)dx \approx \frac{471 + 519}{2} = \boxed{495}$

(13c) $\int_0^6 g(x)dx = \int_0^3 g(x)dx + \int_3^6 g(x)dx \approx 362 + 495 = \boxed{857}$

In problems 15 – 17 , represent the area of each bounded region as a definite integral, and use geometry to
determine the value of the definite integral.

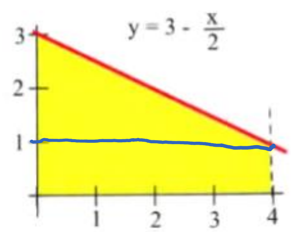
15. The region bounded by $y = 2x$, the x–axis, the line $x = 1$, and $x = 3$.



$$\int_1^2 2x\,dx = \text{ area of rectangle + area of triangle}$$

$$= (2)(2) + \left(\frac{1}{2}\right)(2)(4) = 4 + 4 = \boxed{8}$$

17. The shaded region in the graph to the right.



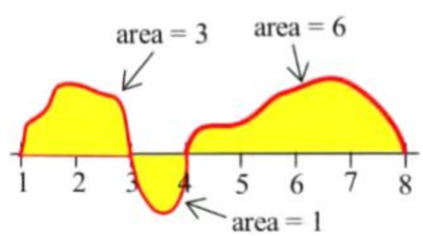
$$\int_0^4 \left(3 - \frac{x}{2}\right)dx = \text{ area of rectangle + area of triangle}$$

$$= (4)(1) + \left(\frac{1}{2}\right)(2)(4) = 4 + 4 = \boxed{8}$$

19. Using the graph of f shown and the given areas of several regions, evaluate:

(a) $\int_1^3 g(x)\,dx$ (b) $\int_3^4 g(x)\,dx$

(c) $\int_4^8 g(x)\,dx$ (d) $\int_1^8 g(x)\,dx$

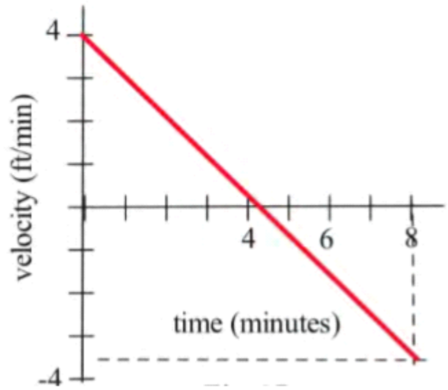


(19a) $\int_1^3 g(x)dx = 3$

(19b) $\int_3^4 g(x)dx = -1$

(19c) $\int_4^8 g(x)dx = 6$

(19d) $\int_1^8 g(x)dx = 3 - 1 + 6 = 8$



21. Your velocity along a straight road is shown to the right. How far did you travel in 8 minutes?

The two triangles have the same area,
so the integral from 0 to 8 of the depicted function is 0.
Thus, the net distance traveled is zero.

However, you could also consider the distance traveled in
going in one direction and then returning to the original position in the opposite direction.
The distance traveled away = $\left(\frac{1}{2}\right)(4\text{min})\left(4\frac{feet}{\text{min}}\right) = 8feet.$
Then, you returned 8 feet.
This means that you traveled 16 feet.

In problems 23 - 26, the units are given for x and for $f(x)$. Give the units of $\int_a^b f(x)\,dx$.

23. x is time in "seconds", and $f(x)$ is velocity in "meters per second."

The units are $\left(\frac{\text{meters}}{\text{second}}\right)(\text{second}) = \boxed{\text{meters}}$

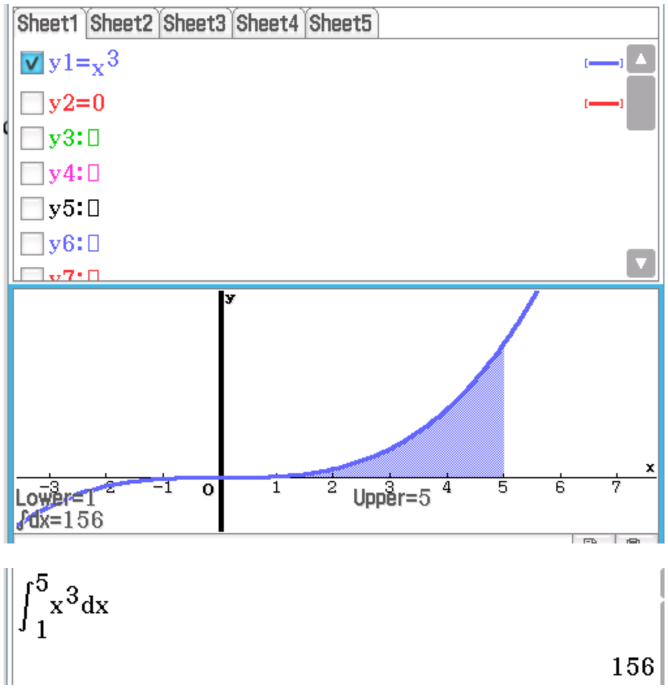
25. x is a position in "feet", and $f(x)$ is an area in "square feet."

The units are $(ft^2)(ft) = \boxed{ft^3}$

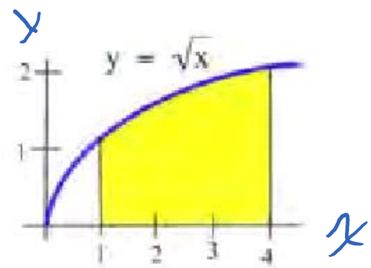
In problems 27 – 31, represent the area with a definite integral and use technology to find the approximate answer.

27. The region bounded by $y = x^3$, the x–axis, the line $x = 1$, and $x = 5$.

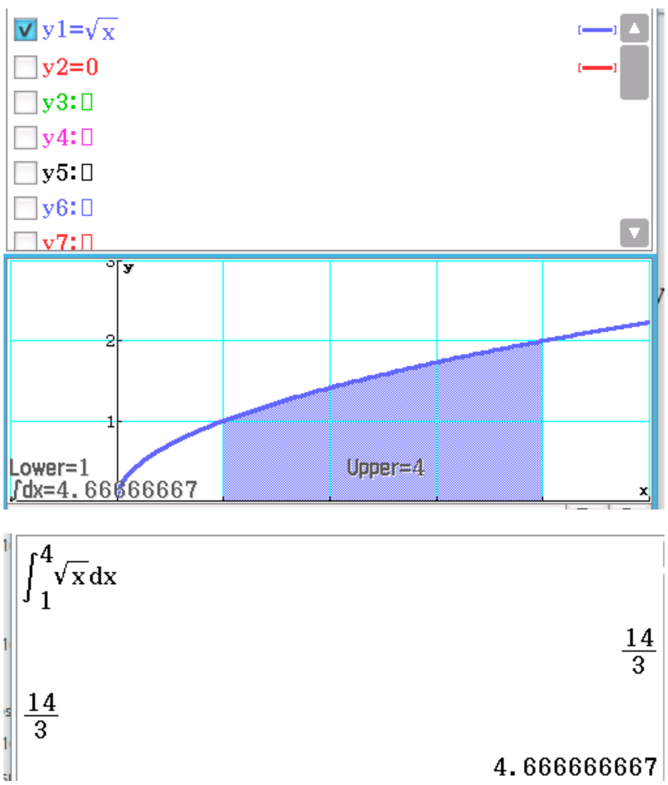
Here is the result with the Casio ClassPad 400, both graphically and as a direct calculation.



29. The shaded region shown to the right.



Here is the result with the Casio ClassPad 400, both graphically and as a direct calculation.

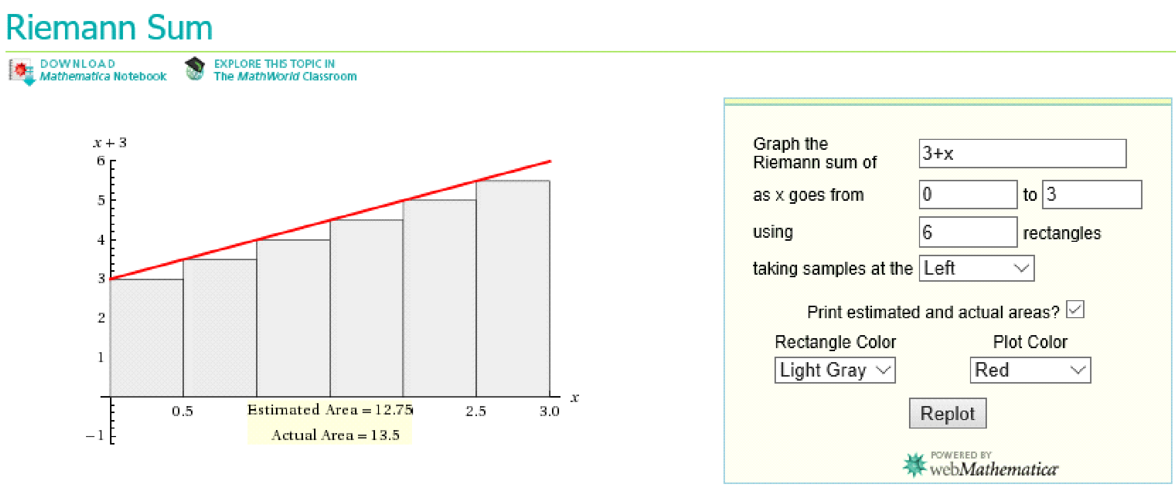


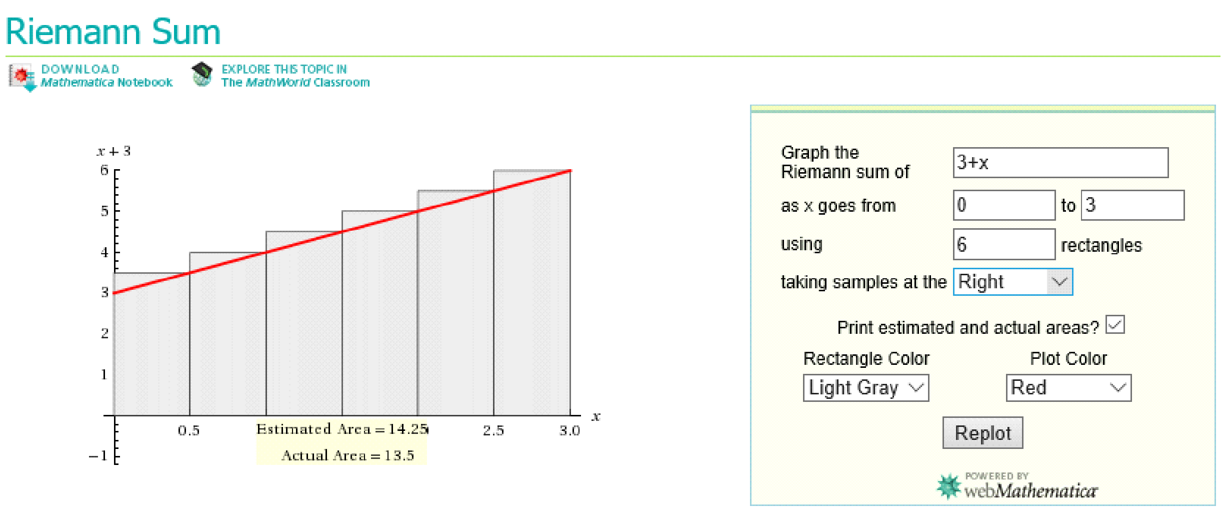
Note that this software gives the exact answer with the direct calculation and a decimal approximation with the graphical calculation.

31. Consider the definite integral $\int_0^3 (3+x)\,dx$.

(a) Using six rectangles, find the left-hand Riemann sum for this definite integral.
(b) Using six rectangles, find the right-hand Riemann sum for this definite integral.
(c) Using geometry, find the exact value of this definite integral.

(31a,b) The following was obtained from Wolfram Mathworld
http://mathworld.wolfram.com/RiemannSum.html





Here is the summation from Casio ClassPad 400.

$$\left(\frac{1}{2}\right)\sum_{i=0}^{5}\left(3+\frac{i}{2}\right)$$

$$\left(\frac{1}{2}\right)\sum_{i=0}^{5}\left(3+\frac{i}{2}\right)$$

12.75

$$\left(\frac{1}{2}\right)\sum_{i=1}^{6}\left(3+\frac{i}{2}\right)$$

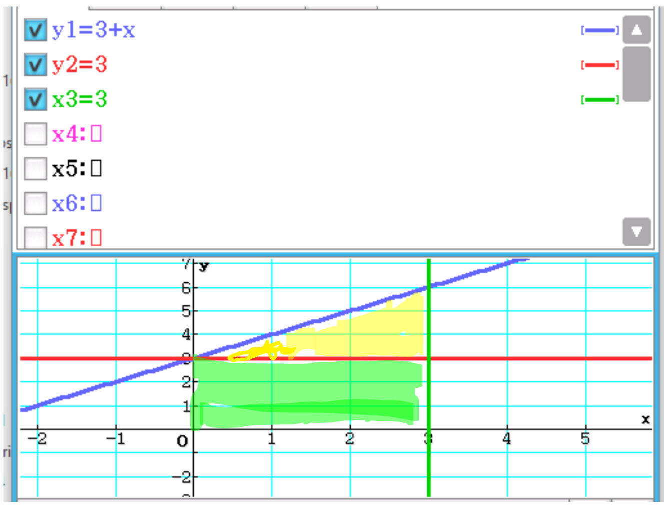
14.25

(31c)

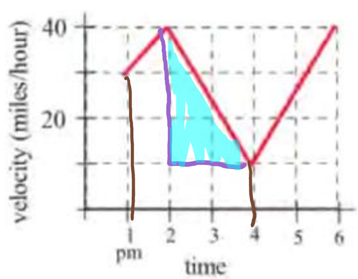
$\int_0^3 (3+x)dx$ = area of rectangle + area of triangle

$$= (3)(3) + \left(\frac{1}{2}\right)(3)(3) = \boxed{\frac{27}{2} = 13.5}$$

Note that $12.75 < 13.5 < 14.25$.



33. Write the total distance traveled by the car in the graph between 1 pm and 4 pm as a definite integral and estimate the value of the integral.



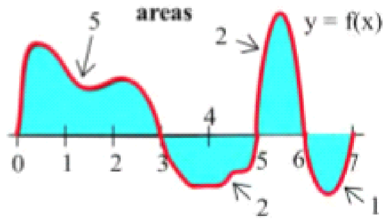
Let the function be v(t),
Where v = velocity at time t.

The distance = $\int_1^4 v(t)dt$

$$\approx \left(5+\frac{1}{2}\right)(1hr)\left(10\frac{mi}{hr}\right)+\left(\frac{1}{2}\right)(2hr)\left(30\frac{mi}{hr}\right)$$

$$= \left(\frac{11}{2}\right)(10mi)+(30mi) = \boxed{85mi}$$

Problems 34 – 41 refer to the graph of f shown. Use the graph to determine the values of the definite integrals. (The bold numbers represent the **area** of each region.)



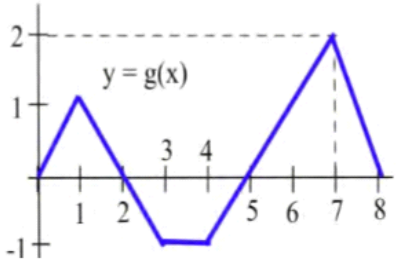
35. $\int_3^5 f(x)\,dx = -2$

37. $\int_6^7 f(w)\,dw = -1$

39. $\int_0^7 f(x)\,dx = 5-2+2-1 = 4$

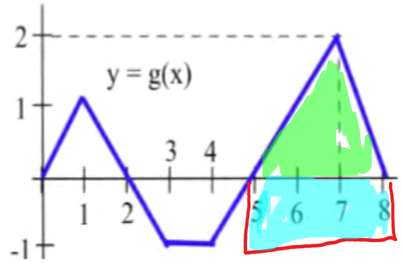
41. $\int_5^7 f(x)\,dx = 2-1=1$

Problems 42 – 47 refer to the graph of g shown. Use the graph to evaluate the integrals.



43. $\int_1^3 g(t)\,dt = \left(\frac{1}{2}\right)(1)(1)-\left(\frac{1}{2}\right)(1)(1)=0$

47. $\int_{5}^{8} 1+g(x)\,dx$



= area of triangle + area of rectangle

$$=\left(\frac{1}{2}\right)(3)(2)+(3)(1)=6$$

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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1* by Shana Calaway, Dale Hoffman, David Lippman