2-8 Applied Calculus Solutions

Tuesday, June 28, 2016 1:21 PM

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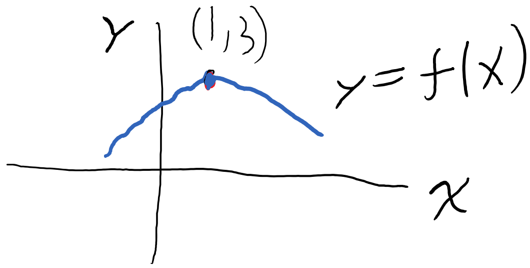
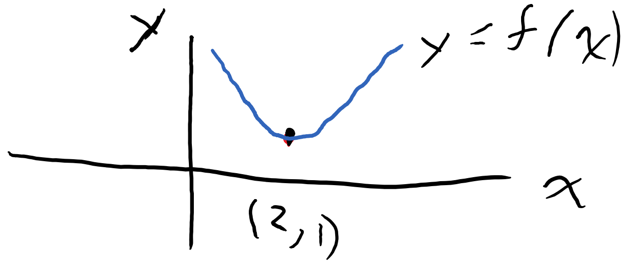
1. Sketch the graph of a continuous function f so that
   (a) $f(1) = 3$, $f'(1) = 0$, and the point (1,3) is a local maximum of f.
   (b) $f(2) = 1$, $f'(2) = 0$, and the point (2,1) is a local minimum of f.
   (c) $f(5) = 4$, $f'(5) = 0$, and the point (5,4) is not a local minimum or maximum of f.

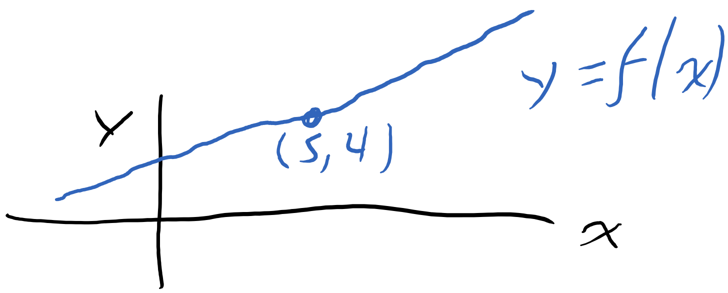
(1a)



(1b)

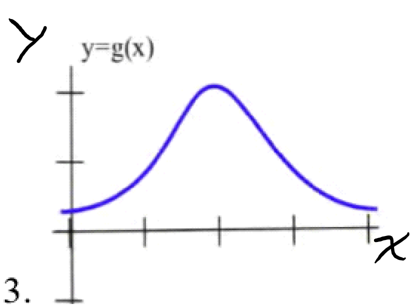


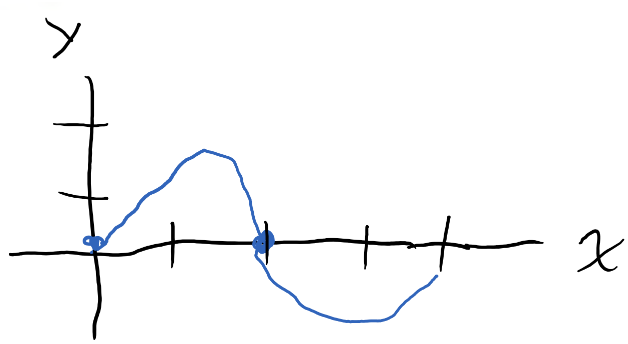
(1c)



In problems 2–4, sketch the graph of the derivative of each function.

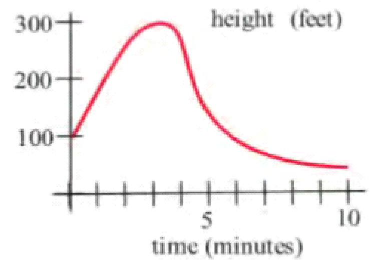
3.

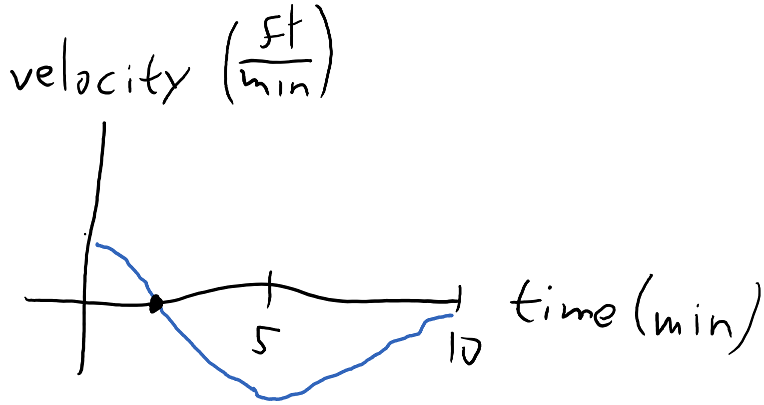




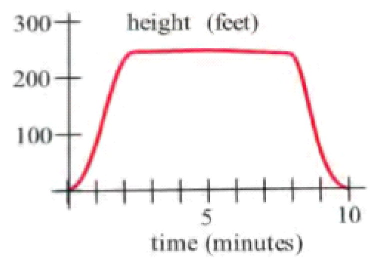
In problems 5–7, the graph of the height of a helicopter is shown. Sketch the graph of the upward velocity of the helicopter.

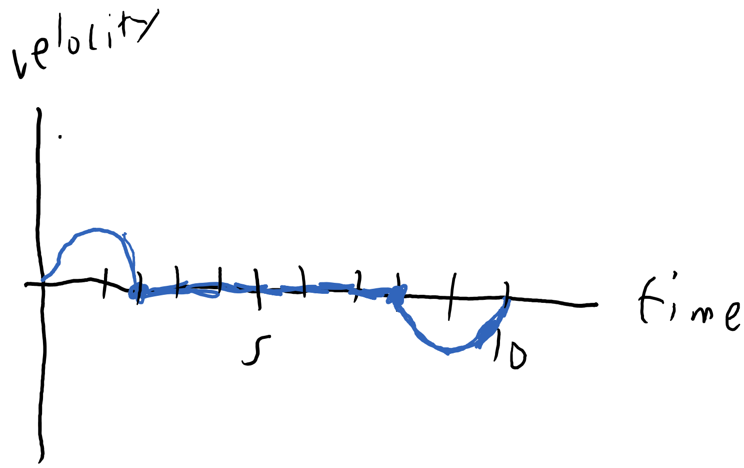
5. 6.



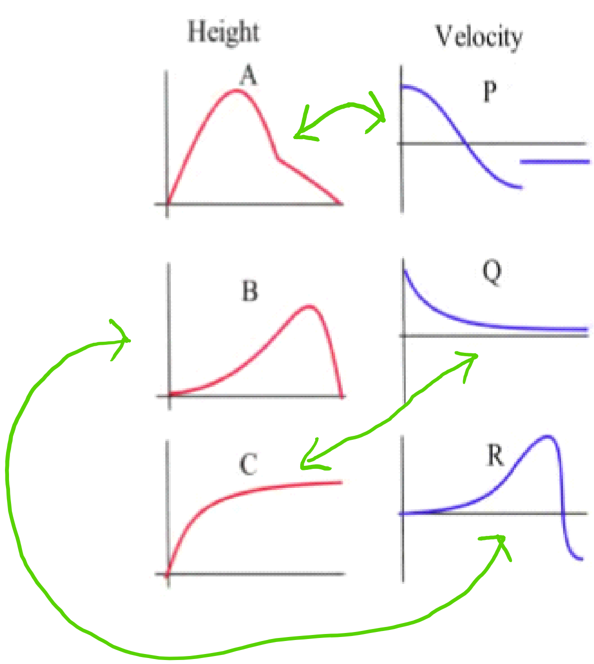


7.





9. In the graphs below, match the graphs showing the heights of rockets with those showing their velocities.



In problems 10 – 14 , use information from the derivatives of each function to help you graph the function. Find all local maximums and minimums of each function.

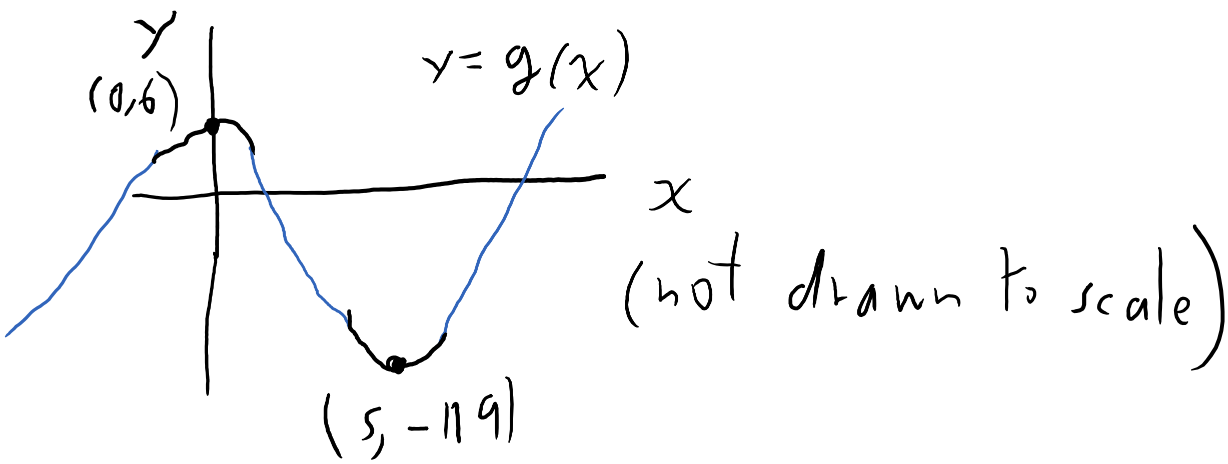
11. $g(x) = 2x^3 - 15x^2 + 6$

$g'(x) = 6x^2 - 30x = 0$
$\Rightarrow x^2 - 5x = 0$
$\Rightarrow x(x-5) = 0$
$\Rightarrow x = 0, 5$

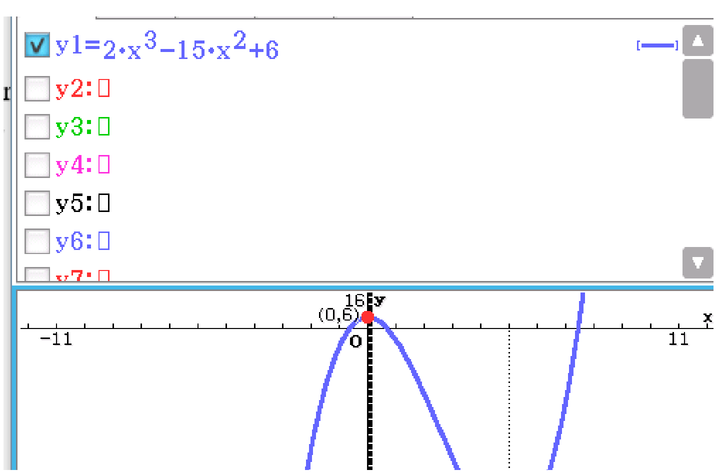
$g''(x) = 12x - 30$
$g''(0) = (12)(0) - 30 = -30 < 0$
$g(0) = 2(0^3) - 15(0^2) + 6 = 6$
$\therefore$ $(0,6)$ is a local maximum

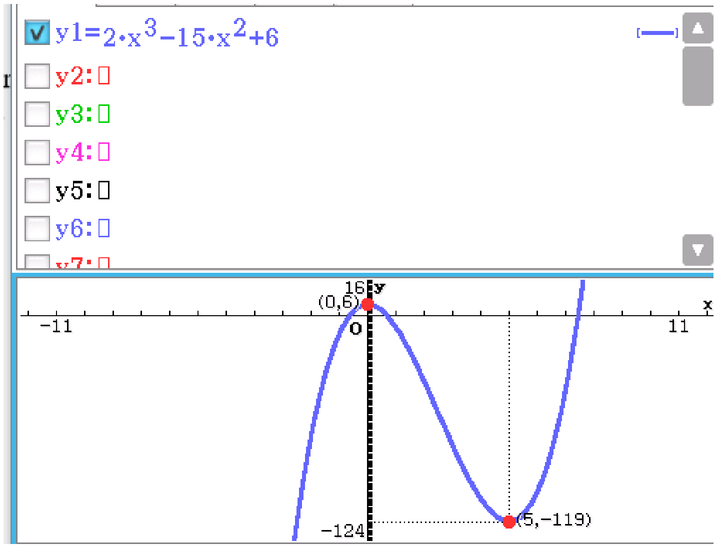
$g''(5) = (12)(5) - 30 = 30 > 0$
$g(5) = 2(5^3) - 15(5^2) + 6 = -119 < 0$
$\therefore$ $(5,-119)$ is a local minimum

We can plot these two points, showing the local "hill" and "valley" and connect the pieces smoothly.



We can graph this with math software to verify our result.





13. $r(t) = \dfrac{2}{t^2+1}$

$r(t) = 2(t^2+1)^{-1}$

$r'(t) = -2(t^2+1)^{-2}(2t) = \boxed{\dfrac{-4t}{(t^2+1)^2} = r'(t)}$

$r'(t)$ is defined for all t.

$r'(t) = 0 \Rightarrow -4t = 0 \Rightarrow t = 0$

$r(0) = 2(0^2+1)^{-1} = 2$

$\therefore$ $(0,2)$ is the only critical point

$$r''(t) = \frac{(t^2+1)^2(-4) - (-4t)(2)(t^2+1)(2t)}{(t^2+1)^4} = \frac{(t^4+2t^2+1)(-4) + 16t^2(t^2+1)}{(t^2+1)^4}$$

$$= \frac{-4t^4 - 8t^2 - 4 + 16t^4 + 16t^2}{(t^2+1)^4} = \frac{12t^4 + 8t^2 - 4}{(t^2+1)^4} = \frac{4(3t^2-1)(t^2+1)}{(t^2+1)^4}$$

$$\boxed{r''(t) = \frac{4(3t^2-1)}{(t^2+1)^3}}$$

$$r''(0) = \frac{4\left(3(0)^2-1\right)}{\left((0)^2+1\right)^3} = -4 < 0$$

$\therefore$ $(0,2)$ is a local maximum

We can use the second derivative to check concavity.

The denominator of $r''(t) > 0$ for all t.

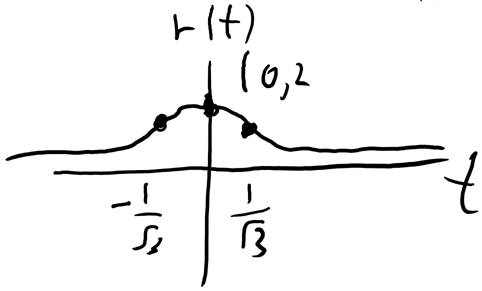
The graph of $r(t)$ is concave up when $r''(t) > 0$ and concave down when $r''(t) < 0$

$$3t^2 - 1 > 0 \Leftrightarrow 3t^2 > 1 \Leftrightarrow t^2 > \frac{1}{3} \Leftrightarrow |t| > \frac{1}{\sqrt{3}} \approx 0.57735$$

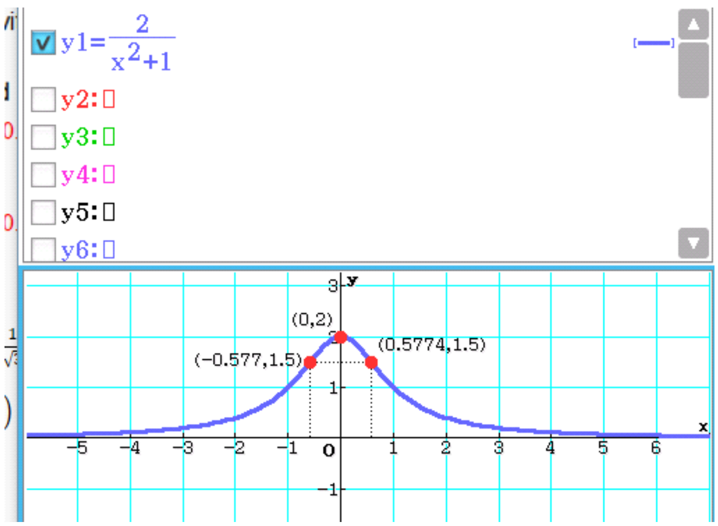
$$3t^2 - 1 < 0 \Leftrightarrow 3t^2 < 1 \Leftrightarrow t^2 < \frac{1}{3} \Leftrightarrow |t| < \frac{1}{\sqrt{3}} \approx 0.57735$$

Thus, the graph is concave down on the interval $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

and concave up on the intervals $\left(-\infty, -\frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{\sqrt{3}}, \infty\right)$.



We can verify this result with math software.



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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1*
by Shana Calaway, Dale Hoffman, David Lippman