2-7 Applied Calculus Solutions

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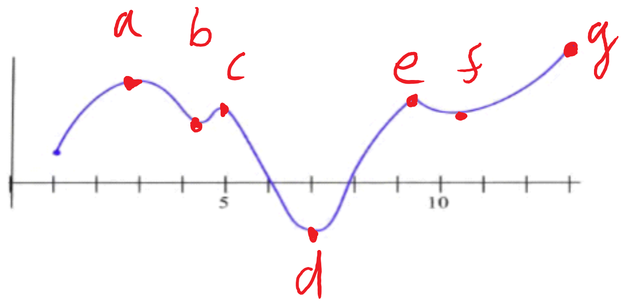
Page 139

1. Find all of the critical points of the function shown and identify them as local max, local min, or neither. Find the global max and min on the interval.



critical points

a - local max
b - local min
c - local max
d - local min
e - local max
f - local min

d - global min
g - global max

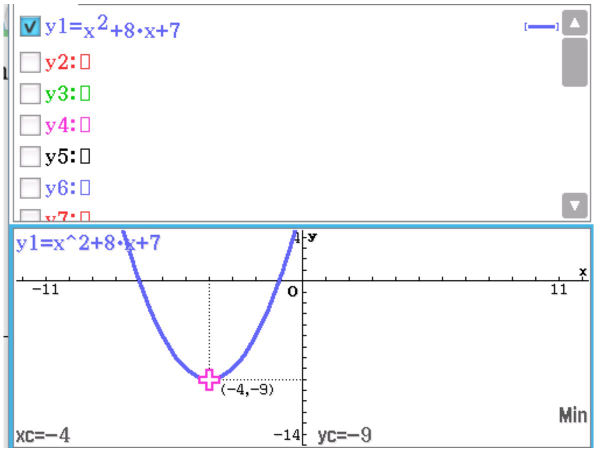
In problems 3 – 8, find all of the critical points and local maximums and minimums of each function.

3. $f(x) = x^2 + 8x + 7$

critical points, where $f'(x) = 0$ or is not defined.
$f'(x) = 2x + 8$, which is defined for all x.
$f'(x) = 0 \Rightarrow 2x + 8 = 0 \Rightarrow 2x = -8 \Rightarrow x = -4$
$f(-4) = (-4)^2 + 8(-4) + 7 = 16 - 32 + 7 = -9$
$\therefore (-4, -9)$ is the only critical point.

$f''(x) = 2 > 0$ for all x$\Rightarrow f''(-4) > 0$
$\therefore (-4, -9)$ is a local minimum.

We can verify this graphically.



5. $f(x) = x^3 - 6x^2 + 5$

critical points, where $f'(x) = 0$ or is not defined.

$f'(x) = 3x^2 - 12x$, which is defined for all x.

$f'(x) = 0 \Rightarrow 3x^2 - 12x = 0 \Rightarrow x(x-4) = 0 \Rightarrow x = 0,4$

$f(0) = (0)^3 - 6(0)^2 + 5 = 5$

$f(4) = (4)^3 - 6(4)^2 + 5 \Rightarrow 64 - 96 + 5 = -27$

The critical points are $(0,5)$ and $(4,-27)$.

$f''(x) = 6x - 12$

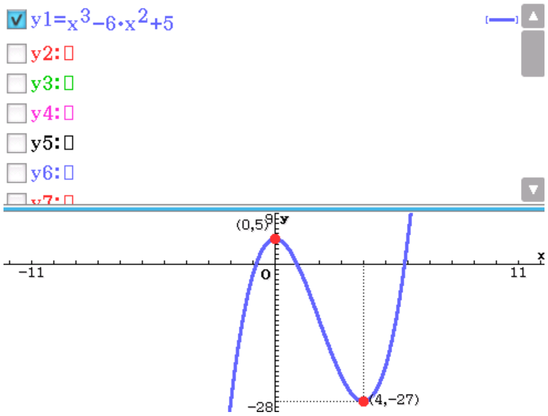
$f''(0) = 6(0) - 12 = -12 < 0$

Thus, $(0,5)$ is a local maximum.

$f''(4) = 6(4) - 12 = 24 - 12 = 12 > 0$

Thus, $(4,-27)$ is a local minimum.

We can verify this graphically.



7. $f(x) = \ln(x^2 - 6x + 11)$

critical points, where $f'(x) = 0$ or is not defined.

$$f'(x) = \frac{1}{x^2-6x+11}\frac{d}{dx}(x^2-6x+11) = \frac{2x-6}{x^2-6x+11}$$

$$x^2 - 6x + 11 = 0 \Rightarrow x = \frac{6 \pm \sqrt{36-44}}{2} = \frac{6 \pm \sqrt{-8}}{2} = \frac{6 \pm 2\sqrt{-2}}{2} = 3 \pm i\sqrt{2}$$

Because there is no real solution, $f'(x)$ is defined for all x.

$$f'(x) = 0 \Rightarrow 2x - 6 = 0 \Rightarrow 2x = 6 \Rightarrow x = 3$$

$$f(3) = \ln(3^2 - 6(3) + 11) = \ln(9 - 18 + 11) = ln(2)$$

Here, the first derivative test will be easier than the second derivative test. The denominator of $f'(x)$ is always positive, so We only need to check the first derivative to the left and to the right of $x = 3$, say at $x = 2$ and $x = 4$.

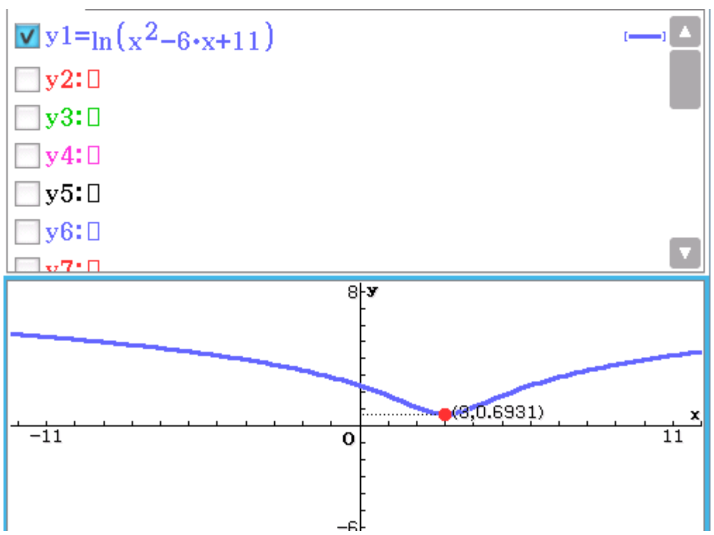
$$2(2) - 6 = 4 - 6 = -2 < 0$$

$$2(4) - 6 = 8 - 6 = 2 > 0$$

Therefore, $(3, \ln(2))$ is a local minimum.

We can verify this graphically.

Note that $\ln(2) \approx 0.693$.



In problems 9 – 16 , find all critical points and global extremes of each function on the given intervals.

9. $f(x) = x^2 - 6x + 5$ on the entire real number line.

$$f'(x) = 2x - 6 = 0 \Rightarrow 2x = 6 \Rightarrow x = 3$$

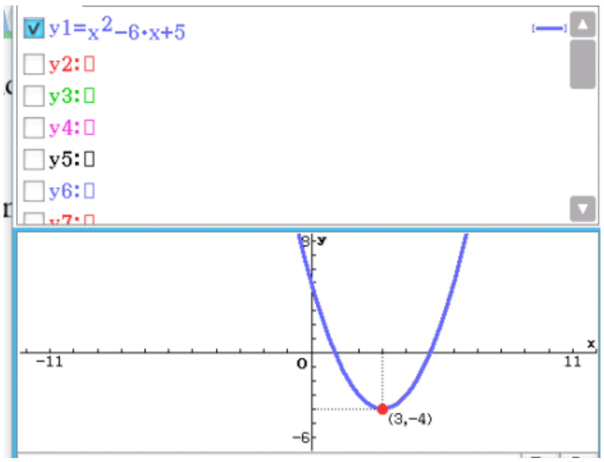
$f(3) = 3^2 - 6(3) + 5 = 9 - 18 + 5 = -4$

Thus, the only critical point is $(3, -4)$.

$f''(x) = 2 > 0$

Thus, $(3, -4)$ is a local minimum.
Since there is no other critical point or end point, this is also a global minimum.
$f(x) \to \infty$ as $x \to \pm\infty$, so there is no global maximum.

We can verify this graphically.



11. f(x) = $x^3 - 3x + 5$ on the entire real number line.

$f'(x) = 3x^2 - 3$, which is defined for all x

$f'(x) = 0 = 3x^2 - 3 \Rightarrow x^2 - 1 = 0 \Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$

$f(-1) = (-1)^3 - 3(-1) + 5 = 7$

$f(1) = (1)^3 - 3(1) + 5 = 3$

The critical points are $(-1, 7)$ and $(1, 3)$.

$f''(x) = 6x$

$\Rightarrow f''(-1) = (6)(-1) = -6 < 0$

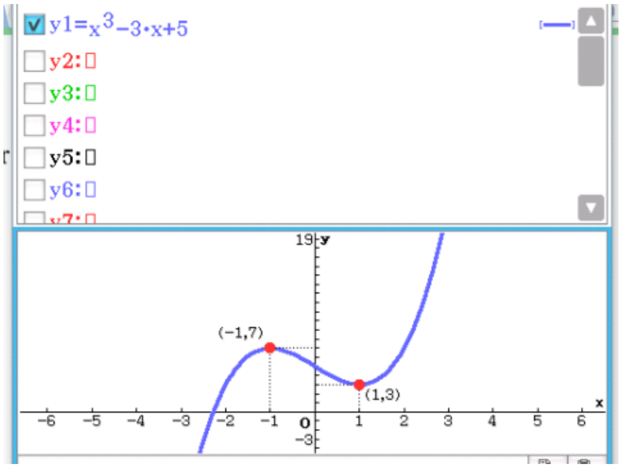
$\Rightarrow$the point $(-1, 7)$ is a local maximum.

$\Rightarrow f''(1) = (6)(1) = 6 > 0$

$\Rightarrow$the point $(1, 3)$ is a local minimum.

There are no endpoints.
$f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$,
so there are no global extremes.

We can verify this graphically.



13. $f(x) = x^2 - 6x + 5$ on $[-2, 5]$.

$f'(x) = 2x - 6$, which is defined for all $x \in [-2, 5]$

$f'(x) = 0 \Rightarrow 2x - 6 = 0 \Rightarrow 2x = 6 \Rightarrow x = 3$
$f(3) == -4$
The only critical point is $(3, -4)$.

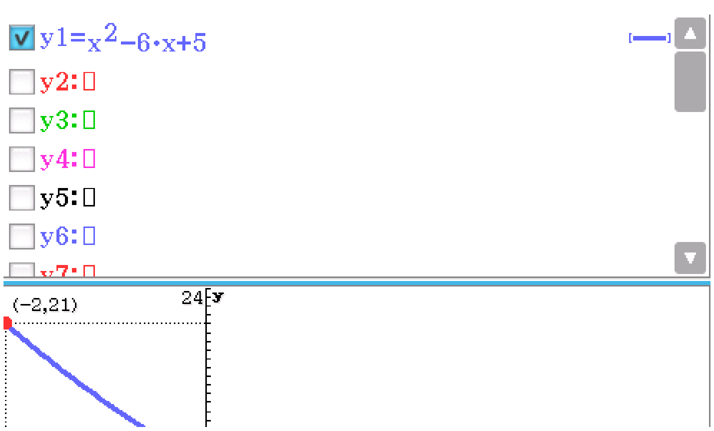
$f''(x) = 2 > 0$ for all x.
$\therefore$ $(3, -4)$ is a local minimum.

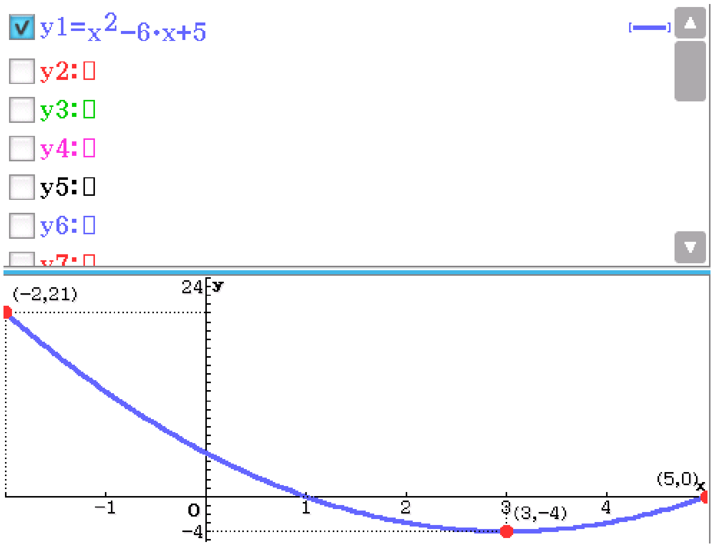
For global extremes, check the endpoints.

$f(-2) = (-2)^2 - (6)(-2) + 5 = 21$
$f(5) = (5)^2 - (6)(5) + 5 = 0$

Comparing the critical point and the endpoints,
we see that the global minimum is $(3, -4)$
and the global maximum is $(-2, 21)$.

We can verify this graphically.





15. f(x) = $x^3 - 3x + 5$ on $[-2, 1]$.

$f'(x) = 3x^2 - 3$, which is defined for all $x \in [-2, 1]$.

$f'(x) = 0 \Rightarrow 3x^2 - 3 = 0 \Rightarrow 3x^2 = 3 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$
Both of these points are in the interval $[-2, 1]$.

$f(-1) = (-1)^3 - (3)(-1) + 5 = 7$
$f(1) = (1)^3 - (3)(1) + 5 = 3$

The critical points are $(-1, 7)$ and $(1, 3)$.

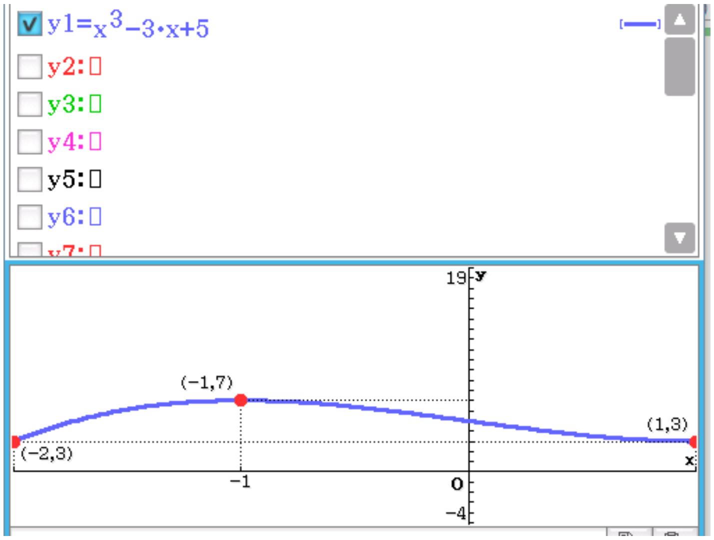
A theorem guarantees the existence of a global maximum and a global minimum. Thus, we need not check for local extremes. We simply compare the critical point with the endpoints.

We have already considered $(1, 3)$.

$f(-2) = (-2)^3 - (3)(-2) + 5 = 3$

The global minimum value is 3, and it occurs at $(-2, 3)$ and $(1, 3)$.
The global maximum is 7 and it occurs at $(-1, 7)$.

We can verify this graphically.



17. Suppose $f(1) = 5$ and $f'(1) = 0$. What can we conclude about the point (1,5) if
 (a) $f'(x) < 0$ for $x < 1$, and $f'(x) > 0$ for $x > 1$?
 (b) $f'(x) < 0$ for $x < 1$, and $f'(x) < 0$ for $x > 1$?
 (c) $f'(x) > 0$ for $x < 1$, and $f'(x) < 0$ for $x > 1$?
 (d) $f'(x) > 0$ for $x < 1$, and $f'(x) > 0$ for $x > 1$?

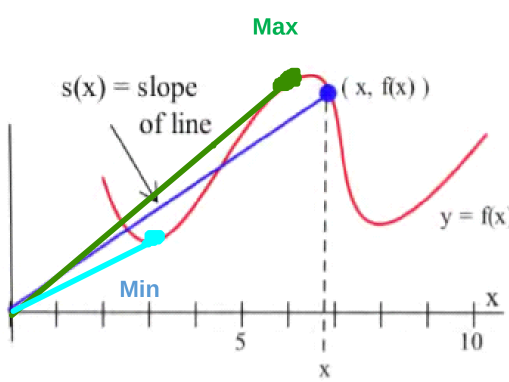
(17a) The point (1,5) is a local minimum.

(17b) The point (1,5) is not a local extreme.

(17c) The point (1,5) is a local maximum.

(17d) The point (1,5) is not a local extreme.

19. Define $S(x)$ to be the **slope** of the line through the points (0,0) and (x, f(x)) based on the graph of f shown.
 (a) At what value of x is $S(x)$ minimum?
 (b) At what value of x is $S(x)$ maximum?

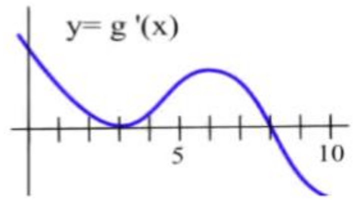


(19a) The aqua line shows the line with minimum slope.
 Here x = 3.

(19b) The green line shows the line with maximum slope.
 Here x =6.

Because we are not given a vertical scale, we cannot calculate, or even estimate the actual slopes.

21. The graph of the **derivative** of a continuous function g.
(a) List the critical numbers of g.
(b) For what values of x does g have a local maximum?
(c) For what values of x does g have a local minimum?



(21a) The critical numbers of g are x = 3 and x = 6.

(21b) g has a local maximum at x = 6.

(21c) g has a local minimum at x = 3.

In problems 22 – 24, a function and values of x so that $f'(x) = 0$ are given. Use the Second Derivative Test to determine whether each point (x, f(x)) is a local maximum, a local minimum or neither

23. $g(x) = x^3 - 3x^2 - 9x + 7$, $x = -1, 3$.

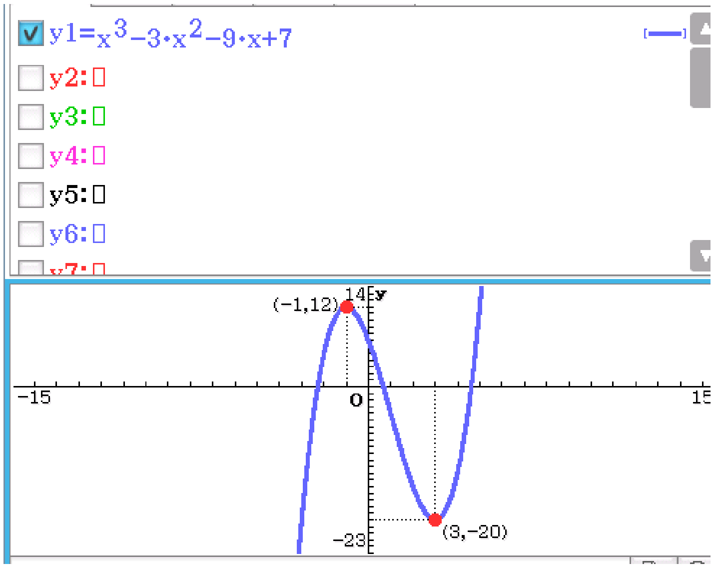
There is a typographical error here. The function f(x) occurs in #22 and h(x) occurs in #24, while g(x) occurs in #23.

$$g'(x) = 3x^2 - 6x - 9$$
$$g''(x) = 6x - 6$$

$$g''(-1) = (6)(-1) - 6 = -12 < 0$$
$$g(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 7 = 12$$
$\therefore$ $(-1, 12)$ is a local maximum.

$$g''(3) = (6)(3) - 6 = 12 > 0$$
$$g(3) = (3)^3 - 3(3)^2 - 9(3) + 7 = -20$$
$\therefore$ $(3, -20)$ is a local minimum.

We can verify this graphically.



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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1*
by Shana Calaway, Dale Hoffman, David Lippman