2-6 Applied Calculus Solutions

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2.6 Exercises

In problems 1 and 2, each quotation is a statement about a quantity of something changing over time. Let f(t) represent the quantity at time t. For each quotation, tell what f represents and whether the first and second derivatives of f are positive or negative.

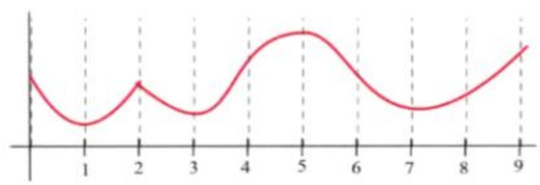
1. (a) "Unemployment rose again, but the rate of increase is smaller than last month."
   (b) "Our profits declined again, but at a slower rate than last month."
   (c) "The population is still rising and at a faster rate than last year."

(1a) f(t) = unemployment rate at month t
f'(t) > 0
f''(t) < 0

(1b) f(t) = profit at month t
f'(t) < 0
f''(t) > 0

(1c) f(t) = population at year t
f'(t) > 0
f''(t) > 0

5. On which intervals is the function in graph
   (a) concave up? (b) concave down?



(5a) function is concave up on $(0,2)$, $(2,4)$, $(6,9)$.

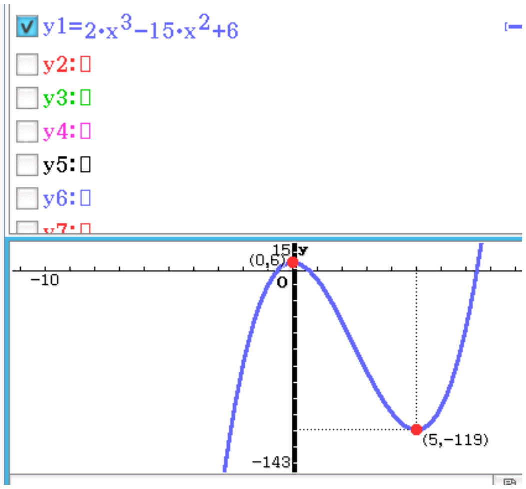
(5b) function is concave down on $(4,6)$.

In problems 7 – 10, a function and values of x so that f '(x) = 0 are given. Use the Second Derivative Test to determine whether each point (x, f(x)) is a local maximum, a local minimum or neither

7. $f(x) = 2x^3 - 15x^2 + 6$, $x = 0, 5$.

$$f'(x) = 6x^2 - 30x$$
$$f''(x) = 12x - 30$$
$$f(0) = 2(0^3) - 15(0^2) + 6 = 6$$
$$f''(0) = (12)(0) - 30 = -30 < 0 \Rightarrow (0,6) \text{ is a local maximum.}$$
$$f(5) = 2(5^3) - 15(5^2) + 6 = 250 - 375 + 6 = -119 < 0$$
$$f''(5) = (12)(5) - 30 = 30 > 0 \Rightarrow (0,-119) \text{ is a local minimum.}$$

Here is a graphical verification of our calculations.

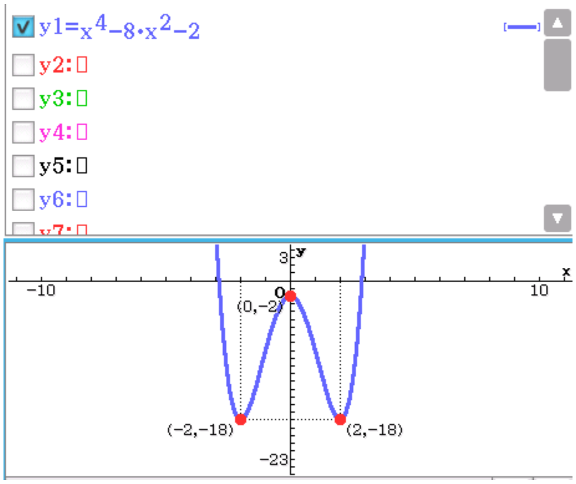


9. $h(x) = x^4 - 8x^2 - 2$, $x = -2, 0, 2$.

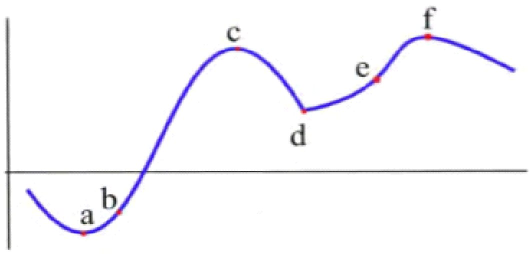
$$h'(x) = 4x^3 - 16x$$
$$h''(x) = 12x^2 - 16$$
$$h(-2) = (-2)^4 - 8(-2)^2 - 2 = 16 - 32 - 2 = -18$$
$$h(0) = (0)^4 - 8(0)^2 - 2 = 0 - 0 - 2 = -2$$
$$h(2) = (2)^4 - 8(2)^2 - 2 = 16 - 32 - 2 = -18$$

$$h''(-2) = 12(-2)^2 - 16 = 48 - 16 = 32 > 0$$
$\Rightarrow (-2, -18)$ is a local minimum.
$$h''(0) = 12(0)^2 - 16 = 0 - 16 = -16 < 0$$
$\Rightarrow (0, -2)$ is a local maximum.
$$h''(2) = 12(2)^2 - 16 = 48 - 16 = 32 > 0$$
$\Rightarrow (-2, -18)$ is a local minimum.

Here is a graphical verification of our calculations.



11. Which of the labeled points in the graph are inflection points?



The point e is definitely an inflection point. While there is an inflection point between point a and point c, the point b appears to be before that unlabeled point.

13. How many inflection points can a (a) quadratic polynomial have? (b) cubic polynomial have? (c) polynomial of degree n have?

(13a) A quadratic polynomial has zero inflection points, because the graph is either always concave up or always concave down.

(13b) A cubic polynomial has one inflection point.

An inflection point occurs where concavity of the function changes. This is where the second derivative is zero or undefined. For a polynomial, all derivatives are defined, so we need only consider where the second derivative is zero.

Let $f(x) = ax^3 + bx^2 + cx + d$ be a cubic polynomial.
$\Rightarrow f'(x) = 3ax^2 + 2bx + c$
$\Rightarrow f''(x) = 6ax + 2b$

$f''(x) = 0 \Leftrightarrow 6ax + 2b = 0 \Leftrightarrow 3ax + b = 0 \Leftrightarrow x = -\frac{b}{3a}$
Because we have a cubic polynomial, we must have $a \neq 0$, so division by a is defined. Depending on the signs of a and b **,** the sign of $f''(x)$ will change from the left to the right of this point. Thus, there is exactly one inflection point.

(13c) A polynomial of degree n has at most $n-2$ inflection points, but it may have fewer.
Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ be a polynomial of degree n.
$\Rightarrow f'(x) = na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + 2a_2x + a_1$
$\Rightarrow f''(x) = n(n-1)a_n x^{n-2} + (n-1)(n-2)a_{n-1}x^{n-3} + \cdots + 2a_2$

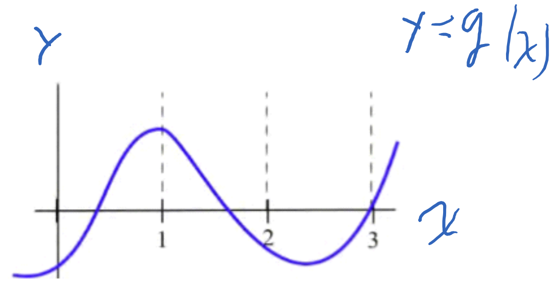
The second derivative is a polynomial of degree $n-2$, which has at most $n-2$ distinct roots, giving at most $n-2$ inflection points.

15. Fill in the table with "+", "–", or "0" for the function shown. Y

Y=9/|x|

15. Fill in the table with "+", "–", or "0" for the function shown.

x	g(x)	g '(x)	g "(x)
0	−	+	+
1	+	−	−
2	−	−	0
3	0	+	+



In problems 16 – 22 , find the derivative and second derivative of each function.

17. $f(x) = (2x - 8)^5$

$$f'(x) = 5(2x-8)^4(2) = 10(2x-8)^4$$

$$f''(x) = (10)(4)(2x-8)^3(2) = 80(2x-8)^3$$

19. $f(x) = x \cdot (3x + 7)^5$

$$f'(x) = x \cdot (5)(3x+7)^4(3) + (3x+7)^5 = (3x+7)^4(15x+3x+7) = (3x+7)^4(18x+7)$$

$$f''(x) = (3x+7)^4(18) + (18x+7)(4)(3x+7)^3(3)$$

$$= (3x+7)^3\left((18)(3x+7) + 12(18x+7)\right)$$

$$= (3x+7)^3\left(54x + 126 + 216x + 84\right)$$

$$= (3x+7)^3\left(270x + 210\right)$$

$$= 30(3x+7)^3\left(9x+7\right)$$

21. $f(x) = \sqrt{x^2 + 6x - 1}$

$$f(x) = (x^2 + 6x - 1)^{1/2}$$

$$f'(x) = \left(\frac{1}{2}\right)(x^2+6x-1)^{-1/2}(2x+6) = \frac{x+3}{\sqrt{x^2+6x-1}}$$

$$f''(x) = \left(x^2+6x-1\right)^{-1/2}(1) + \left(x+3\right)\left(\frac{1}{2}\right)\left(x^2+6x-1\right)^{-3/2}(2x+6)$$

$$= \left(x^2+6x-1\right)^{-1/2} + \left(x+3\right)^2\left(x^2+6x-1\right)^{-3/2}$$

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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1*
by Shana Calaway, Dale Hoffman, David Lippman