2-5 Applied Calculus Solutions

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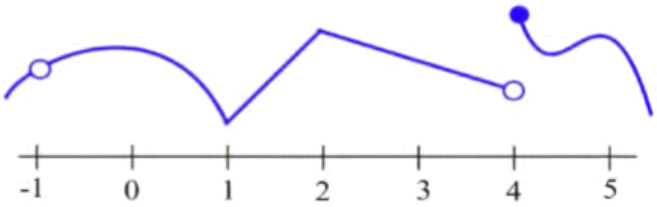
1. The graph of $y = f(x)$ is shown.
   (a) At which integers is f continuous?
   (b) At which integers is f differentiable?



(1a) f is continuous at all integers except $x = -1$ and $x = 4$.

(1b) f is differentiable at all integers except $x = -1$,1,2,4.

Problems 3 and 4 refer to the values given in this table:

x	f(x)	g(x)	f '(x)	g '(x)	(f∘g)(x)	(f∘g)' (x)
−2	2	−1	1	1		
−1	1	2	0	2		
0	−2	1	2	−1		
1	0	−2	−1	2		
2	1	0	1	−1		

3. Use the table of values to determine (f∘g)(x) and (f∘g)' (x) at x = 1 and 2.

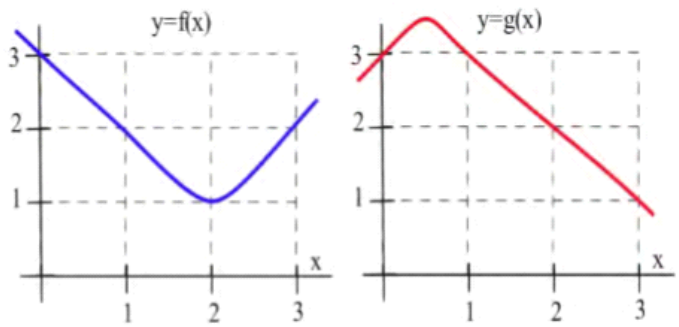
$(f \circ g)(1) = f\big(g(1)\big) = f(-2) = \boxed{2}$

$(f \circ g)(2) = f\big(g(2)\big) = f(0) = \boxed{-2}$

$(f \circ g)'(1) = f'(g(1)) \cdot g'(1) = f'(-2) \cdot (2) = (1)(2) = \boxed{2}$

$(f \circ g)'(2) = f'(g(2)) \cdot g'(2) = f'(0) \cdot (-1) = (2)(-1) = \boxed{-2}$

5. Use the graphs to estimate the values of g(x), **g '(x)**, (f∘g)(x), **f '(g(x))**, and **(f∘g) '(x)** at x = 1.



$g(1) \approx 3$

$g'(1) \approx -1$

$(f \circ g)(1) = f(g(1)) \approx f(3) \approx 2$

$f'(g(1)) \approx f'(3) \approx 1$

$(f \circ g)'(1) = f'(g(1))\, g'(1) \approx (1)(-1) = -1$

In problems 7 – 12 , find the derivative of each function.

7. $f(x) = (2x - 8)^5$

$$f'(x) = 5(2x-8)^4 \frac{d}{dx}(2x-8) = 5(2x-8)^4(2) = \boxed{10(2x-8)^4}$$

9. $f(x) = x \cdot (3x + 7)^5$

$$f'(x) = x\frac{d}{dx}(3x+7)^5 + (3x+7)^5\frac{d}{dx}(x) = x(5)(3x+7)^4\frac{d(3x+7)}{dx}$$
$$= 5x(3x+7)^4(3) + (3x+7)^5$$
$$= 15x(3x+7)^4 + (3x+7)^5$$
$$= (3x+7)^4(15x+3x+7)$$
$$= \boxed{(3x+7)^4(18x+7)}$$

11. $f(x) = \sqrt{x^2 + 6x - 1}$

$$f(x) = (x^2+6x-1)^{1/2}$$
$$\Rightarrow f'(x) = \left(\frac{1}{2}\right)(x^2+6x-1)^{1/2-1}\frac{d}{dx}(x^2+6x-1)$$
$$= \left(\frac{1}{2}\right)(x^2+6x-1)^{-1/2}(2x+6)$$
$$= \boxed{(x+3)(x^2+6x-1)^{-1/2} = \frac{(x+3)}{\sqrt{x^2+6x-1}}}$$

13. If f is a differentiable function,
    (a) how are the graphs of $y = f(x)$ and $y = f(x) + k$ related?
    (b) how are the derivatives of $f(x)$ and $f(x) + k$ related?

The graph of $y = f(x) + k$ is the graph of $y = f(x)$ shifted vertically k units.
The derivatives of $f(x)$ and $f(x) + k$ are equal.

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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1* by Shana Calaway, Dale Hoffman, David Lippman

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