2-4 Applied Calculus Solutions

Wednesday, June 8, 2016 7:42 PM

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1. Use the values in the table to fill in the rest of the table.

x	f(x)	f '(x)	g(x)	g '(x)	$\frac{d}{dx}(f(x)\cdot g(x))$	$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$	$\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right)$
0	3	–2	–4	3	17	-1/16	1/9
1	2	–1	1	0	-1	-1	1/4
2	4	2	3	1	10	2/9	-1/8

$$\frac{d}{dx}(f(x)\cdot g(x)) = f(x)g'(x) + g(x)f'(x)$$

$$\frac{d}{dx}(f(0)\cdot g(0)) = f(0)g'(0) + g(0)f'(0) = (3)(3) + (-4)(-2) = 17$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}\left(\frac{f(0)}{g(0)}\right) = \frac{g(0)f'(0) - f(0)g'(0)}{(g(0))^2} = \frac{(-4)(-2)-(3)(3)}{(-4)^2} = \frac{-1}{16}$$

$$\frac{d}{dx}\left(\frac{g(x)}{f(x)}\right) = \frac{f(x)g'(x) - g(x)f'(x)}{(f(x))^2}$$

$$\frac{d}{dx}(f(x)\cdot g(x)) = \frac{d}{dx}(k)$$

5. If the product of f and g is a constant ($f(x)\cdot g(x) = k$ for all x), then how are $\dfrac{\frac{d}{dx}(f(x))}{f(x)}$ and $\dfrac{\frac{d}{dx}(g(x))}{g(x)}$ related?

$$\Rightarrow f(x)\cdot g'(x) + f'(x)\cdot g(x) = 0$$

$$\Rightarrow f(x)\cdot g'(x) = -f'(x)\cdot g(x)$$

$$\Rightarrow \frac{g'(x)}{g(x)} = -\frac{f'(x)}{f(x)}$$

$$\Rightarrow \boxed{\frac{\frac{d(g(x))}{dx}}{g(x)} = -\frac{\frac{d(f(x))}{dx}}{f(x)}}$$

In problems 7 – 8, (a) calculate f '(1) and (b) determine when f '(x) = 0.

7. $f(x) = \dfrac{7x}{x^2 + 4}$

(7a)

$$f'(x) = \frac{(x^2+4)\frac{d}{dx}(7x) - (7x)\frac{d}{dx}(x^2+4)}{(x^2+4)^2}$$

$$= \frac{(x^2+4)(7) - (7x)(2x)}{(x^2+4)^2} = \frac{(7x^2+28) - 14x^2}{(x^2+4)^2}$$

$$= \boxed{\frac{28 - 7x^2}{(x^2+4)^2}}$$

$$f'(1) = \frac{28 - 7(1^2)}{(1^2+4)^2} = \boxed{\frac{21}{25}}$$

(7b)

$$f'(x) = 0 \Rightarrow \frac{28 - 7x^2}{(x^2+4)^2} = 0$$

$$\Rightarrow 28 - 7x^2 = 0 \Rightarrow 7x^2 = 28$$

$$\Rightarrow x^2 = \frac{28}{7} = 4$$

$$\Rightarrow \boxed{x = \pm 2}$$

9. Determine $\dfrac{d}{dx}(x^2+1)(7x-3)$ and $\dfrac{d}{dt}\left(\dfrac{3t-2}{5t+1}\right)$.

$$\frac{d}{dx}(x^2+1)(7x-3) = (x^2+1)\frac{d}{dx}(7x-3) + (7x-3)\frac{d}{dx}(x^2+1)$$

$$= (x^2+1)(7) + (7x-3)(2x)$$

$$= 7x^2 + 7 + 14x^2 - 6x$$

$$= \boxed{21x^2 - 6x + 7}$$

$$\frac{d}{dt}\left(\frac{3t-2}{5t+1}\right) = \frac{(5t+1)\frac{d(3t-2)}{dt} - (3t-2)\frac{d}{dt}(5t+1)}{(5t+1)^2}$$

$$= \frac{(5t+1)(3)-(3t-2)(5)}{(5t+1)^2} = \frac{15t+3-(15t-10)}{(5t+1)^2}$$

$$= \frac{15t+3-15t+10}{(5t+1)^2} = \boxed{\frac{13}{(5t+1)^2}}$$

11. Find (a) $\frac{d}{dt}\left(te^t\right)$, (b) $d\left(e^x\right)^5$

(11a)

$$\frac{d}{dt}(te^t) = t\frac{d}{dt}e^t + e^t\frac{d}{dt}t$$

$$= te^t + e^t(1)$$

$$= \boxed{e^t(t+1)}$$

(11b) This could refer to something called the differential, but that is not introduced here, so the problem should be to find $\frac{d}{dx}(e^x)^5$.
However, this requires the chain rule, which will be introduced in section 2.5. Otherwise, we would need a tedious application of the product rule.

$$\frac{d}{dx}(e^x)^5 = \frac{d}{dx}e^{5x}$$

$$= e^{5x}\frac{d}{dx}(5x) = \boxed{5e^{5x}}$$

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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1* by Shana Calaway, Dale Hoffman, David Lippman ⇒