2-3 Applied Calculus Solutions

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1. Fill in the values in the table for $\frac{d}{dx}(3f(x))$, $\frac{d}{dx}(2f(x)+g(x))$, and $\frac{d}{dx}(3g(x)-f(x))$.

x	f(x)	f '(x)	g(x)	g '(x)	$\frac{d}{dx}(3f(x))$	$\frac{d}{dx}(2f(x)+g(x))$	$\frac{d}{dx}(3g(x)-f(x))$
0	3	–2	–4	3	–6	–1	11
1	2	–1	1	0	–3	–2	1
2	4	2	3	1	6	5	1

$D(x^{-4})$

3. Find (a) $D(x^9)$ (b) $\frac{d\,x^{2/3}}{dx}$ (c) $D(\frac{1}{x^4})$ (d) $D(x^\pi)$

(3a) $D(x^9) = 9x^8$

(3b) $\frac{d}{dx}\left(x^{\frac{2}{3}}\right) = \left(\frac{2}{3}\right)x^{\frac{2}{3}-1} = \left(\frac{2}{3}\right)x^{-\frac{1}{3}}$

(3c) $D\left(\frac{1}{x^4}\right) = D(x^{-4}) = -4x^{-4-1} = -4x^{-5}$

(3d) $D(x^\pi) = \pi x^{\pi-1}$

In problems 4 – 8, (a) calculate f '(1) and (b) determine when f '(x) = 0.

5. $f(x) = 5x^2 - 40x + 73$

(5a)

$f'(x) = 10x - 40$

$\Rightarrow f'(1) = (10)(1) - 40 = -30$

(5b)

$10x - 40 = 0$

$\Rightarrow 10x = 40$

$\Rightarrow x = \frac{40}{10}$

$\Rightarrow x = 4$

7. $f(x) = x^3 + 3x^2 + 3x - 1$

(7a) $f'(x) = 3x^2 + 6x + 3$

$\Rightarrow f'(1) = 3(1^2) + 6(1) + 3 = 12$

(7b) $3x^2 + 6x + 3 = 0$
$\Rightarrow x^2 + 2x + 1 = 0$
$\Rightarrow (x+1)^2 = 0$
$\therefore x = -1$

9. Where do $f(x) = x^2 - 10x + 3$ and $g(x) = x^3 - 12x$ have horizontal tangent lines ?

In each case, set the first derivative equal to 0 and solve for x.

$f(x) = x^2 - 10x + 3 \Rightarrow f'(x) = 2x - 10$

$2x - 10 = 0 \Rightarrow 2x = 10 \Rightarrow x = 5$

Thus, the graph of $y = f(x)$ has a horizontal tangent line at $x = 5$.

$g(x) = x^3 - 12x \Rightarrow g'(x) = 3x^2 - 12$

$3x^2 - 12 = 0 \Rightarrow x^2 - 4 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

Thus, the graph of $y = g(x)$ has a horizontal tangent line at $x = \pm 2$.

11. It costs $C(x) = \sqrt{x}$ dollars to produce x golf balls. What is the marginal production cost to make a golf ball? What is the marginal production cost when x = 25? when x= 100? (Include units.)

The marginal production cost is the first derivative of the cost.

$$C(x) = \sqrt{x} = x^{1/2}$$
$$\Rightarrow C'(x) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$
$$C'(25) = \frac{1}{2\sqrt{25}} = \frac{1}{(2)(5)} = \frac{1}{10}$$

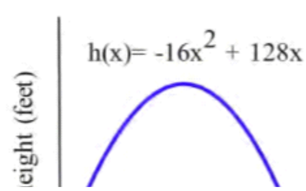
Therefore the marginal production cost to make a golf ball when producing 25 golf balls is 10 cents.

$$C'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{(2)(10)} = \frac{1}{20}$$

Therefore the marginal production cost to make a golf ball when producing 25 golf balls is 5 cents.

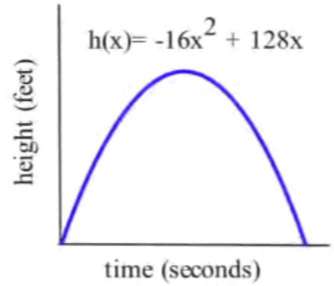
12. An arrow shot straight up from ground level with an initial velocity of 128 feet per second will be at height $h(x) = -16x^2 + 128x$ feet at x seconds.

(a) Determine the velocity of the arrow when x = 0, 1 and 2 seconds.
(b) What is the velocity of the arrow, v(x), at any time x?
(c) At what time x will the velocity of the arrow be 0?
(d) What is the greatest height the arrow reaches?



12. An arrow shot straight up from ground level with an initial velocity of 128 feet per second will be at height $h(x) = -16x^2 + 128x$ feet at x seconds.

(a) Determine the velocity of the arrow when $x = 0$, 1 and 2 seconds.
(b) What is the velocity of the arrow, $v(x)$, at any time x?
(c) At what time x will the velocity of the arrow be 0?
(d) What is the greatest height the arrow reaches?
(e) How long will the arrow be aloft?
(f) Use the answer for the velocity in part (b) to determine the acceleration, $a(x) = v'(x)$, at any time x.



(12a) velocity = $h'(x) = -32x + 128$

$$h'(0) = -32(0) + 128 = \frac{128ft}{sec}$$

$$h'(1) = -32(1) + 128 = \frac{96ft}{sec}$$

$$h'(2) = -32(2) + 128 = \frac{64ft}{sec}$$

(12b)

$$v(x) = h'(x) = (-32x + 128)\frac{ft}{sec}$$

(12c) $v(x) = 0 \Rightarrow -32x + 128 = 0 \Rightarrow 32x = 128 \Rightarrow x = \frac{128}{32} = 4$

Therefore, the velocity will be 0 when $x = 4sec$.

(12d) The height will be maximum when the velocity = 0.
The maximum height = $h(4) = -16(4^2) + 128(4) = -256 + 512 = 256ft$

(12e) The arrow will be aloft between the two times that the height = 0.

$$h(x) = 0$$
$$\Rightarrow -16(x^2) + 128(x) = 0$$
$$\Rightarrow x^2 - 8x = 0$$
$$\Rightarrow x(x - 8) = 0$$
$$\therefore x = 0 \text{ and } x = 8$$

The arrow will be aloft for 8 seconds.

(12f) acceleration = $v'(x) = \frac{d}{dx}(-32x + 128) = -32\frac{ft}{\sec^2}$

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