2-2 Applied Calculus Solutions

Thursday, June 2, 2016 3:23 PM

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1. What is the slope of the line through (3,9) and (x, y) for $y = x^2$ and $x = 2.97$? $x = 3.001$? $x = 3+h$? What happens to this last slope when h is very small (close to 0)? Sketch the graph of $y = x^2$ for x near 3.

At $x = 2.97$, $y = 2.97^2 = 8.8209$.

slope $= \dfrac{8.8209 - 9}{2.97 - 3} = \boxed{5.97}$

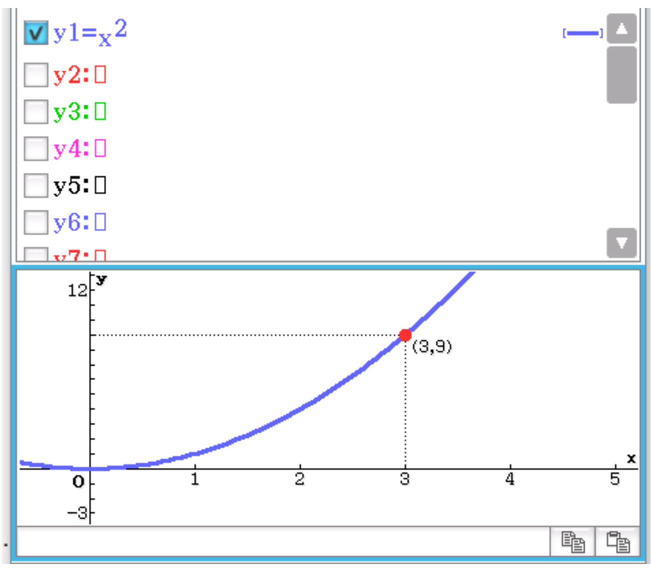
At $x = 3.001$, $y = 3.001^2 = 9.006$.

slope $= \dfrac{9.006 - 9}{3.001 - 3} = \boxed{6.0}$

At $x = 3 + h$, $y = (3 + h)^2 = h^2 + 6h + 9$

slope $= \dfrac{(h^2 + 6h + 9) - 9}{(3 + h) - 3} = \dfrac{h^2 + 6h}{h} = \boxed{6 + h}$, if $h \neq 0$

As h approaches 0, then slope approaches $6 + 0 = 6$.



3. What is the slope of the line through (2,4) and (x, y) for $y = x^2 + x - 2$ and $x = 1.99$? $x = 2.004$? $x = 2+h$? What happens to this last slope when h is very small? Sketch the graph of $y = x^2 + x - 2$ for x near 2.

At $x = 1.99$, $y = 1.99^2 + 1.99 - 2 = 3.9501$

slope = $\dfrac{3.9501 - 4}{1.99 - 2} = \boxed{4.99}$

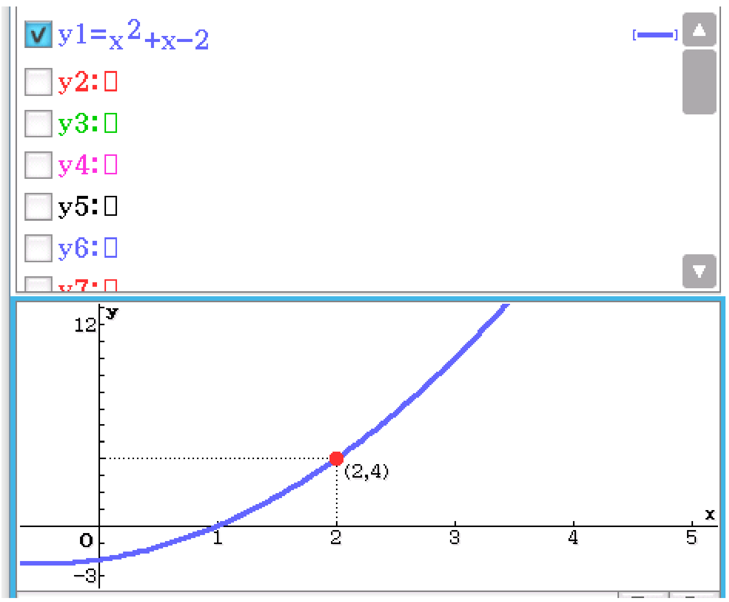
At $x = 2.004$, $y = 2.004^2 + 2.004 - 2 = 4.02$

slope = $\dfrac{4.02 - 4}{2.004 - 2} = \boxed{5.0}$

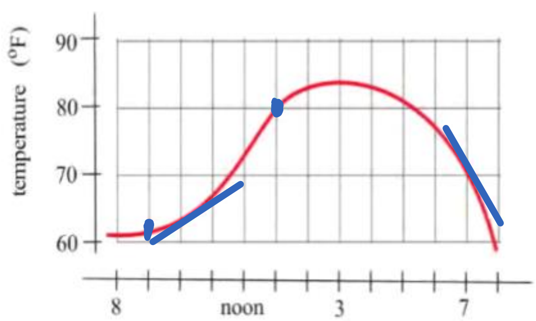
At $x = 2 + h$, $y = (2+h)^2 + (2+h) - 2 = h^2 + 5h + 4$

slope = $\dfrac{(h^2 + 5h + 4) - 4}{(2+h) - 2} = \dfrac{h^2 + 5h}{h} = 5 + h$, if $h \neq 0$

As h approaches 0, then slope approaches $5 + 0 = \boxed{5}$.



5. The graph to the right shows the temperature during a day in Ames.
   (a) What was the average change in temperature from 9 am to 1 pm?
   (b) Estimate how fast the temperature was rising **at** 10 am and **at** 7 pm?



(5a)

average change in temperature =

$$\frac{\text{change in temperature}}{\text{change in time}} = \frac{80^\circ\text{F} - 62^\circ\text{F}}{1\text{pm} - 9\text{am}} = \boxed{\frac{18^\circ\text{F}}{4\,\text{h}}}$$

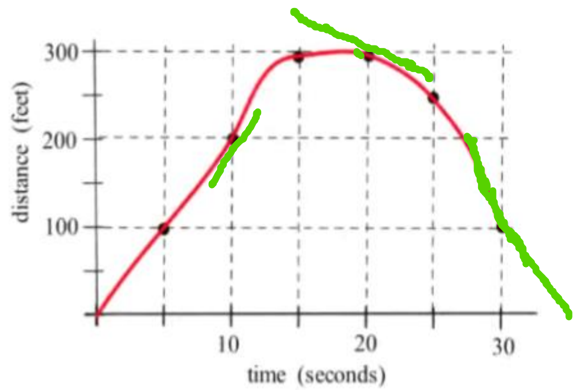
(5b) From the rough sketch of the tangent line at 10 am, we can estimate the slope of the tangent line to obtain an estimate of how fast the temperature was rising at 10 am.

$$\text{slope} \approx \frac{9\,^\circ\text{F}}{3\,\text{h}} = \boxed{\frac{3\,^\circ\text{F}}{\text{h}}}$$

Likewise, at 7 pm,

$$\text{slope} \approx \frac{-18\,^\circ\text{F}}{2\,\text{h}} = \boxed{\frac{-9\,^\circ\text{F}}{\text{h}}}$$

7. The graph shows the distance of a car from a measuring position located on the edge of a straight road.
   (a) What was the average velocity of the car from $t = 0$ to $t = 20$ seconds?
   (b) What was the average velocity from $t = 10$ to $t = 30$ seconds?
   (c) About how fast was the car traveling **at** $t = 10$ seconds? **at** $t = 20$ s ? **at** $t = 30$ s ?



(7a) average velocity $= \dfrac{\Delta\,\text{distance}}{\Delta\,\text{time}} = \dfrac{300ft - 0ft}{20\text{sec} - 0\text{sec}} = \dfrac{300ft}{20\text{sec}} = \boxed{\dfrac{15ft}{\text{sec}}}$

(7b) average velocity $= \dfrac{\Delta\,\text{distance}}{\Delta\,\text{time}} = \dfrac{100ft - 200ft}{30\text{sec} - 10\text{sec}} = \dfrac{-100ft}{20\text{sec}} = \boxed{\dfrac{-5ft}{\text{sec}}}$

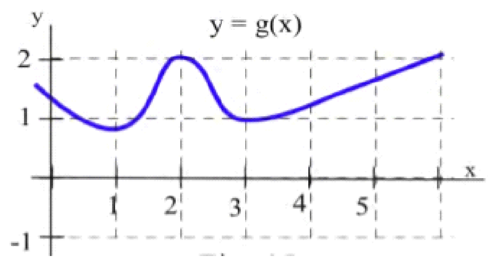
(7c) the speed of the car at a given time = slope of tangent line at that point.

$$t = 10\text{sec} \Rightarrow speed \approx \frac{100ft}{5\text{sec}} = \boxed{\frac{20ft}{\text{sec}}}$$

$$t = 20\text{sec} \Rightarrow speed \approx \frac{-50ft}{10\text{sec}} = \boxed{\frac{-5ft}{\text{sec}}}$$

$$t = 30\text{sec} \Rightarrow speed \approx \frac{-200ft}{10\text{sec}} = \boxed{\frac{-20ft}{\text{sec}}}$$

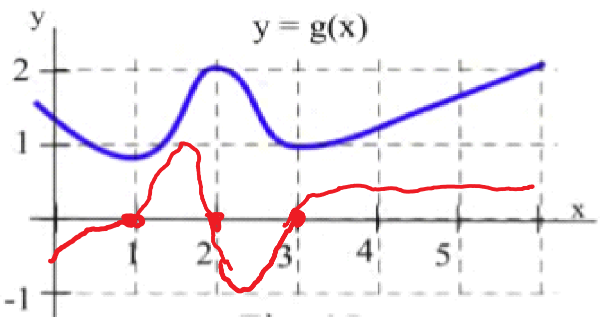
13. (a) At what values of x does the graph of g have a horizontal tangent line?
(b) At what value(s) of x is the value of g the largest? smallest?
(c) Sketch the graph of m(x) = the slope of the line tangent to the graph of g at the point (x,y).



(13a) The graph of g has a horizontal tangent line at $x = 1,2,3$.

(13b) The value of g is largest at $x = 2,6$.
The value of g is smallest at $x = 2,1$.

(13c)



For each function f(x) in problems 15 – 20, perform steps (a) – (d):

(a) Calculate $m_{\sec} = \frac{f(x+h)-f(x)}{h}$ and simplify

(b) determine $m_{\tan} = \lim_{h\to 0} m_{\sec}$

(c) evaluate $m_{\tan}$ at $x = 2$

(d) find the equation of the line tangent to the graph of f at $(2, f(2))$.

15. f(x) = 3x – 7

(15a) $m_{\sec} = \frac{f(x+h)-f(x)}{h} = \frac{[3(x+h)-7]-[3x-7]}{h} = \frac{3x+3h-7-3x+7}{h} = \frac{3h}{h} = \boxed{3}$

(15b) $m_{\tan} = \lim_{h\to 0} m_{\sec} = \lim_{h\to 0} 3 = \boxed{3}$

(15c) at $x = 2$, $m_{\tan} = \boxed{3}$

17. f(x) = ax + b where a and b are constants

(17a)

$$m_{\sec} = \frac{f(x+h) - f(x)}{h} = \frac{[\mathrm{a}(x+h) + \mathrm{b}] - [\mathrm{a}x + \mathrm{b}]}{h} = \frac{\mathrm{a}x + \mathrm{a}h + \mathrm{b} - \mathrm{a}x - \mathrm{b}}{h} = \frac{\mathrm{a}h}{h} = \boxed{\mathrm{a}}$$

(17b) $m_{\tan} = \lim_{h\to 0} m_{\sec} = \lim_{h\to 0} a = \boxed{\mathrm{a}}$

(17c) at $x = 2$, $m_{\tan} = \boxed{\mathrm{a}}$

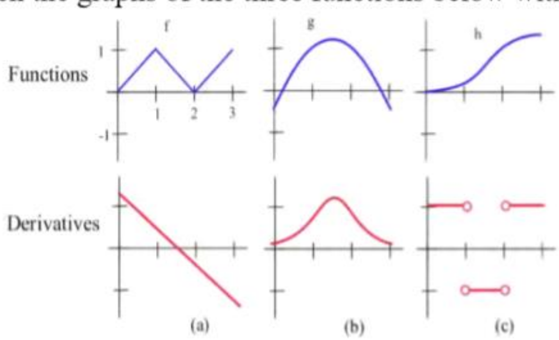
19. $f(x) = 8 - 3x^2$

(19a) $m_{\sec} = \frac{f(x+h)-f(x)}{h} = \frac{[8-3(x+h)^2]-[8-3x^2]}{h} = \frac{8-3x^2-6xh-3h^2-8+3x^2}{h} = \frac{-6xh-3h^2}{h} = -6x - 3h$

(19b) $m_{\tan} = \lim_{h\to 0} m_{\sec} = \lim_{h\to 0}(-6x - 3h) = -6x$

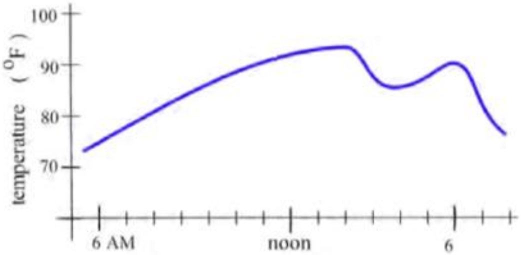
(19c) At $x = 2$, $m_{\tan} = (-6)(2) = -12$

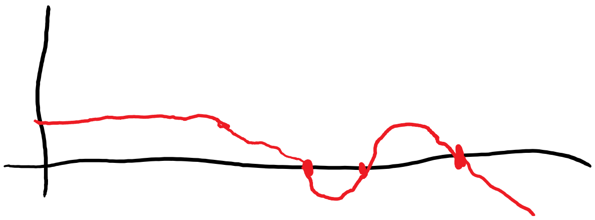
21. Match the graphs of the three functions below with the graphs of their derivatives.



g h f

23. The graph below shows the temperature during a summer day in Chicago. Sketch the graph of the **rate** at which the temperature is changing. (This is just the graph of the **slopes** of the lines which are tangent to the temperature graph.)





25. If $C(x)$ is the total cost, in millions, of producing x thousand items, interpret $C'(4) = 2$.

When 4000 items are produced, the cost of producing another 1000 items will be $2 million.

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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1*
by Shana Calaway, Dale Hoffman, David Lippman