2-11 Applied Calculus Solutions

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In problems 1 – 10 find dy/dx by differentiating implicitly then find the value of dy/dx at the given point.

1. $x^2 + y^2 = 100$, point (6, 8)

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(100)$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$x + y\frac{dy}{dx} = 0$$

$$y\frac{dy}{dx} = -x$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

$$\frac{dy}{dx}(6,8) = \frac{-6}{8} = \boxed{-\frac{3}{4}}$$

3. $x^2 - 3xy + 7y = 5$, point (2,1)

$$\frac{d}{dx}(x^2) - \frac{d}{dx}(3xy) + \frac{d}{dx}(7y) = \frac{d}{dx}(5)$$

$$2x - 3x\frac{dy}{dx} - 3y + 7\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(-3x + 7) = -2x + 3y$$

$$\boxed{\frac{dy}{dx} = \frac{-2x + 3y}{-3x + 7}}$$

$$\frac{dy}{dx}(2,1) = \frac{-2(2) + 3(1)}{-3(2) + 7} = \boxed{-1}$$

5. $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$, point (0,4)

$$\frac{d}{dx}\left(\frac{x^2}{9}\right) + \frac{d}{dx}\left(\frac{y^2}{16}\right) = \frac{d}{dx}(1)$$

$$\frac{2x}{9} + \frac{2y}{16}\frac{dy}{dx} = 0$$

$$\frac{y}{8}\frac{dy}{dx} = -\frac{2x}{9}$$

$$\frac{dy}{dx} = -\left(\frac{2x}{9}\right)\left(\frac{8}{y}\right) = \boxed{-\frac{16x}{y}}$$

$$\frac{dy}{dx}(0,4) = -\frac{16(0)}{4} = \boxed{0}$$

7. $\ln(y) + 3x - 7 = 0$, point (2,e)

$$\frac{d}{dx}(\ln(y)) + \frac{d}{dx}(3x) - \frac{d}{dx}(7) = \frac{d}{dx}(0)$$

$$\left(\frac{1}{y}\right)\frac{dy}{dx} + 3 - 0 = 0$$

$$\left(\frac{1}{y}\right)\frac{dy}{dx} = -3$$

$$\boxed{\frac{dy}{dx} = -3y}$$

$$\boxed{\frac{dy}{dx}(2,e) = -3e}$$

9. $x^2 - y^2 = 16$, point (5, –3)

$$2x - 2y\frac{dy}{dx} = 0$$

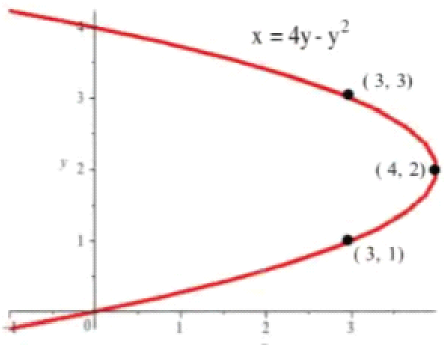
$$x - y\frac{dy}{dx} = 0$$

$$y\frac{dy}{dx} = x$$

$$\boxed{\frac{dy}{dx} = \frac{x}{y}}$$

$$\boxed{\frac{dy}{dx}(5,-3) = \frac{5}{-3} = -\frac{5}{3}}$$

11. Find the slopes of the lines tangent to the graph in shown at the points $(3,1)$, $(3,3)$, and $(4,2)$.



$$x = 4y - y^2$$

$$\frac{d}{dx}(x) = \frac{d}{dx}(4y) - \frac{d}{dx}(y^2)$$

$$1 = 4\frac{dy}{dx} - 2y\frac{dy}{dx}$$

$$\frac{dy}{dx}(4-2y) = 1$$

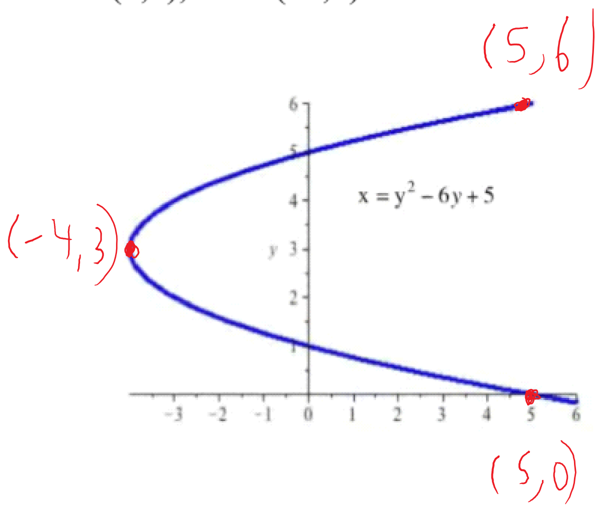
$$\boxed{\frac{dy}{dx} = \frac{1}{4-2y}}$$

$$\frac{dy}{dx}(3,1) = \frac{1}{4-2(1)} = \boxed{\frac{1}{2}}$$

$$\frac{dy}{dx}(3,3) = \frac{1}{4-2(3)} = \boxed{-\frac{1}{2}}$$

$$\frac{dy}{dx}(4,2) = \frac{1}{4-2(2)} = \frac{1}{0}$$, which is not defined.

13. Find the slopes of the lines tangent to the graph in graph shown at the points ((5,0), (5,6), and (–4,3).



$$x = y^2 - 6y + 5$$

$$\frac{d}{dx}(x) = \frac{d}{dx}(y^2) - \frac{d}{dx}(6y) + \frac{d}{dx}(5)$$

$$1 = 2y\frac{dy}{dx} - 6\frac{dy}{dx} + 0$$

$$2y\frac{dy}{dx} - 6\frac{dy}{dx} = 1$$

$$y\frac{dy}{dx} - 3\frac{dy}{dx} = \frac{1}{2}$$

$$\frac{dy}{dx}(y-3) = \frac{1}{2}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{2(y-3)}}$$

$$\frac{dy}{dx}(5,0) = \frac{1}{2(0-3)} = \boxed{-\frac{1}{6}}$$

$$\frac{dy}{dx}(5,6) \quad 1 \quad \boxed{1}$$

$$\frac{dy}{dx}(5,0) = \frac{1}{2(0-3)} = \boxed{-\frac{1}{6}}$$

$$\frac{dy}{dx}(5,6) = \frac{1}{2(6-3)} = \boxed{\frac{1}{6}}$$

$$\frac{dy}{dx}(-4,3) = \frac{1}{2(3-3)} = \frac{1}{0}$$, which is not defined.

In problems 15 – 16, find dy/dx using implicit differentiation and then find the slope of the line tangent to the graph of the equation at the given point.

15. $y^3 - 5y = 5x^2 + 7$, point (1,3)

$$y^3 - 5y = 5x^2 + 7$$

$$\frac{d}{dx}(y^3) - \frac{d}{dx}(5y) = \frac{d}{dx}(5x^2) + \frac{d}{dx}(7)$$

$$3y^2\frac{dy}{dx} - 5\frac{dy}{dx} = 10x + 0$$

$$\frac{dy}{dx}(3y^2 - 5) = 10x$$

$$\boxed{\frac{dy}{dx} = \frac{10x}{3y^2 - 5}}$$

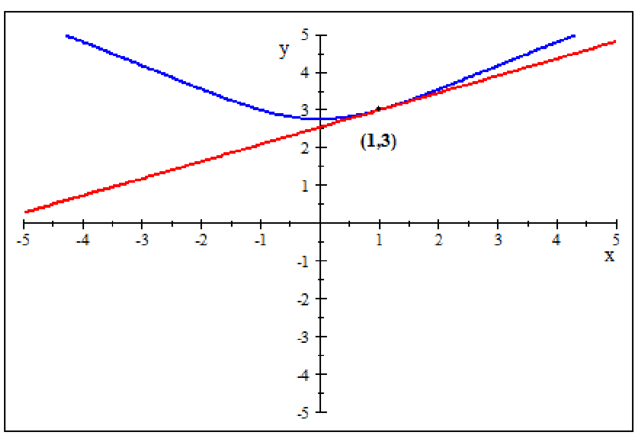
$$\frac{dy}{dx}(1,3) = \frac{10(1)}{3(3)^2 - 5} = \boxed{\frac{5}{11}}$$

$\therefore$ The slope of the tangent line at $(1,3)$ is $\frac{5}{11}$.

We can verify our calculations by graphing the curve implicitly in Scientific Notebook 5.5, along with the tangent line and the point of tangency.

$y^3 - 5y = 5x^2 + 7$

tangent line at $(1,3)$ is $y = \frac{5x}{11} + \frac{28}{11}$



17. An expandable sphere is being filled with liquid at a constant rate from a tap (imagine a water balloon connected to a faucet). When the radius of the sphere is 3 inches, the radius is increasing at 2 inches per minute. How fast is the liquid coming out of the tap? ($V = \frac{4}{3}\pi r^3$)

Let V = volume of sphere in cubic inches
Let r = radius of sphere in inches
Let t = time in minutes

$V = \frac{4\pi r^3}{3}$

When $r = 3\,\text{in}$, $\frac{dr}{dt} = \frac{2\,\text{in}}{\text{min}}$

The volume of water coming out of the tap per minute = the increase in volume of the sphere per minute.

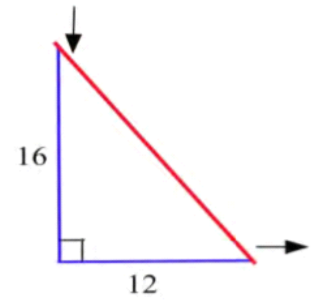
Find $\frac{dV}{dt}$.

$$\frac{dV}{dt} = \frac{d\left(\frac{4\pi r^3}{3}\right)}{dt} = \frac{4\pi(3r^2)}{3}\frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi(3\,\text{in})^2\left(\frac{2\,\text{in}}{\text{min}}\right) = \boxed{\frac{72\pi\,\text{in}^3}{\text{min}} \approx \frac{226.19\,\text{in}^3}{\text{min}}}$$

By carrying the units throughout the calculation and arriving at the units of $\frac{\text{volume}}{\text{time}}$, we can be confident that the calculation was set up properly.

18. The 12 inch base of a right triangle is growing at 3 inches per hour, and the 16 inch height is shrinking at 3 inches per hour.
 (a) Is the area increasing or decreasing?
 (b) Is the perimeter increasing or decreasing?
 (c) Is the hypotenuse increasing or decreasing?



Let $x(t)$ = base of triangle in inches at time t.
Let t = time in hours.
Let $y(t)$ = height of triangle in inches at time t.

When $x = 12\,\text{in}$ and $y = 16\,\text{in}$, $\frac{dx}{dt} = \frac{3\,\text{in}}{\text{h}}$ and $\frac{dy}{dt} = \frac{-3\,\text{in}}{\text{h}}$

Let $A(t)$ = area of triangle at time t.
Let $P(t)$ = perimeter of triangle at time t.
Let $h(t)$ = hypotenuse of triangle at time t.

(18a) $A = \left(\frac{1}{2}\right)xy$

$$\Rightarrow \frac{dA}{dt} = \left(\frac{1}{2}\right)\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right) = \left(\frac{1}{2}\right)\left((12\,\text{in})\left(\frac{-3\,\text{in}}{\text{h}}\right) + (16\,\text{in})\left(\frac{3\,\text{in}}{\text{h}}\right)\right)$$

$$= (-18 + 48)\frac{\text{in}^2}{\text{h}} = \boxed{30\,\frac{\text{in}^2}{\text{h}}} > 0$$

$\therefore$ The area is increasing.

(18b) $P(t) = x + y + \sqrt{x^2 + y^2} = x + y + (x^2 + y^2)^{1/2}$

$$\frac{dP}{dt} = \frac{dx}{dt} + \frac{dy}{dt} + \left(\frac{1}{2}\right)(x^2 + y^2)^{-1/2}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$$

$$= \frac{3\,\text{in}}{\text{h}} + \frac{-3\,\text{in}}{\text{h}} + \left(\frac{1}{2}\right)\left((12\,\text{in})^2 + (16\,\text{in})^2\right)^{-1/2}\left((2)(12\,\text{in})\left(\frac{3\,\text{in}}{\text{h}}\right) + (2)(16\,\text{in})\left(-\frac{3\,\text{in}}{\text{h}}\right)\right)$$

$$= \left((12\,\text{in})^2 + (16\,\text{in})^2\right)^{-1/2}\left((12\,\text{in})\left(\frac{3\,\text{in}}{\text{h}}\right) + (16\,\text{in})\left(-\frac{3\,\text{in}}{\text{h}}\right)\right)$$

$$= (144\,\text{in}^2 + 256\,\text{in}^2)^{-1/2}\left(\frac{36}{\text{h}}\,\text{in}^2 + -\frac{48}{\text{h}}\,\text{in}^2\right)$$

$$= (400\,\text{in}^2)^{-1/2}\left(-\frac{12}{\text{h}}\,\text{in}^2\right)$$

$$= \frac{1}{20\,\text{in}}\left(-\frac{12}{\text{h}}\,\text{in}^2\right) = \boxed{-\frac{3\,\text{in}}{5\,\text{h}}} < 0$$

Therefore, the perimeter is decreasing.

(18c) $h(t) = \sqrt{x^2 + y^2}$

$$P(t) = x + y + \sqrt{x^2 + y^2} = x(t) + y(t) + h(t)$$

$$\frac{dP}{dt} = \frac{dx}{dt} + \frac{dy}{dt} + \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{dP}{dt} - \frac{dx}{dt} - \frac{dy}{dt}$$

$$= -\frac{3\text{ in}}{5\text{ h}} - \frac{3\text{ in}}{\text{h}} - \left(\frac{-3\text{ in}}{\text{h}}\right) = \boxed{-\frac{3\text{ in}}{5\text{ h}}} < 0$$

∴ The hypotenuse is decreasing.

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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1* by Shana Calaway, Dale Hoffman, David Lippman