2-10 Applied Calculus Solutions

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1. If $g(20) = 35$ and $g'(20) = -2$, estimate the value of $g(22)$.

$$g(22) \approx g(20) + g'(20) \cdot (22 - 20)$$
$$= 35 - 2(2) = 35 - 4 = \boxed{31}$$

3. Use the Tangent Line Approximation to estimate the cube root of 9.

Let $f(x) = x^{1/3}$
We estimate $f(9)$.
We search for the nearest perfect cube, which is $8 = 2^3$.

$$f(9) \approx f(8) + f'(8) \cdot (9 - 8)$$
$$f(8) = 8^{1/3} = 2$$

$$f'(x) = \left(\frac{1}{3}\right)x^{-2/3} \Rightarrow f'(8) = \left(\frac{1}{3}\right)(8)^{-2/3} = \left(\frac{1}{3}\right)\frac{1}{\left(\sqrt[3]{8}\right)^2}$$
$$= \left(\frac{1}{3}\right)\frac{1}{(2)^2} = \frac{1}{12}$$
$$f(9) \approx 2 + \frac{1}{12} \cdot (1) = \frac{25}{12} \approx \boxed{2.0833}$$

We can check this by using a calculator or math software.

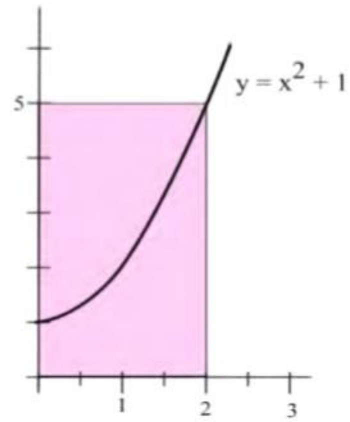
$$\boxed{\sqrt[3]{9} \approx 2.0801}$$

This shows that are estimate is accurate to two decimal places.

5. A rectangle has one side on the x–axis, one side on the y–axis, and a corner on the graph of $y = x^2 + 1$.

(a) Use Linear Approximation of the area formula to estimate the increase in the area of the rectangle if the base grows from 2 to 2.3 inches.

(b) Calculate exactly the increase in the area of the rectangle as the base grows from 2 to 2.3 inches.



(5a)

Let $A(x)$ = area of rectangle with x = coordinate of right endpoint of the base

$A(x) = xy = x(x^2+1) = x^3 + x$

We will use dA to estimate $\triangle A = A(2.3) - A(2)$.

$dA = A'(2) \cdot (2.3 - 2)$

$A'(x) = 3x^2 + 1 \Rightarrow A'(2) = 3(2^2) + 1 = 13$

$dA = (13) \cdot (0.3) = 3.9$

$\boxed{\therefore \triangle A \approx dA = 3.9}$

(5b)

$A(2.3) - A(2) = (2.3^3 + 2.3) - (2^3 + 2) = 4.467$

$\therefore \boxed{\triangle A = 4.467}$

Our answer is plausible, because the curve $y = x^2 + 1$ increases more rapidly than the tangent line at $x = 2$.

7. The demand function for Alicia's oven mitts is given by $q = -8p + 80$ (q is the number of oven mitts, p is the price in dollars). Find the elasticity of demand when $p = \$7.50$. Will revenue increase if Alicia raises her price from \$7.50?

elasticity of demand $= E = \left| \dfrac{p}{q} \cdot \dfrac{dq}{dp} \right|$

$p = 7.50 \Rightarrow q = -8(7.50) + 80 = 20.0$

$q = -8p + 80 \Rightarrow \dfrac{dq}{dp} = -8$

$\Rightarrow E = \left| \dfrac{7.5}{20} \cdot (-8) \right| = |-3.0| = 3.0$

$\therefore$ elasticity of demand is 3

If **E < 1,** we say demand is **inelastic.** In this case, raising prices increases revenue.
If **E > 1**, we say demand is **elastic.** In this case, raising prices decreases revenue.
If **E = 1**, we say demand is **unitary**. E = 1 at critical points of the revenue function.

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Because $3 > 1$, raising the price will decrease the revenue.
We can see why this is so by a direct calculation.

The revenue $R = pq = p(-8p + 80) = 80p - 8p^2$

$\dfrac{dR}{dp} = 80 - 16p$

$\dfrac{dR}{dp}(7.5) = 80 - (16)(7.50) = -40.0 < 0$

Thus, the revenue does decrease if the price is raised above $\$7.50$.

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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1*
by Shana Calaway, Dale Hoffman, David Lippman