1-8 Applied Calculus Solutions

Sunday, May 29, 2016 6:25 PM

Page 72

Rewrite each equation in exponential form

1. $\log(v) = t$

$10^t = v$

3. $\ln(w) = n$

$e^n = w$

Rewrite each equation in logarithmic form.

5. $10^a = b$

$\log(b) = a$

7. $e^k = h$

$\ln(h) = k$

Solve each equation for the variable.

9. $5^x = 14$

$\ln(5^x) = \ln(14)$

$x\ln(5) = \ln(14)$

$x = \frac{\ln(14)}{\ln(5)} \approx 1.64$

11. $7^x = \frac{1}{15}$

$\ln(7^x) = \ln\left(\frac{1}{15}\right)$

$\Rightarrow x\ln(7) = \ln\left(\frac{1}{15}\right) = -\ln(15)$

$\Rightarrow \boxed{x = \frac{-\ln(15)}{\ln(7)} \approx -1.39}$

13. $e^{5x} = 17$

$$\ln(e^{5x}) = \ln(17)$$
$$\Rightarrow 5x = \ln(17)$$
$$\Rightarrow \boxed{x = \frac{\ln(17)}{5} \approx 0.567}$$

15. $3^{4x-5} = 38$

$$\ln(3^{4x-5}) = \ln(38)$$
$$\Rightarrow (4x-5)\ln(3) = \ln(38)$$
$$\Rightarrow 4x\ln(3) = 5\ln(3) + \ln(38)$$
$$\Rightarrow \boxed{x = \frac{5\ln(3)+\ln(38)}{4\ln(3)} \approx 2.078}$$

17. $1000(1.03)^t = 5000$

$$(1.03)^t = \frac{5000}{1000} = 5$$
$$\Rightarrow \ln\left((1.03)^t\right) = \ln(5)$$
$$\Rightarrow t\ln(1.03) = \ln(5)$$
$$\Rightarrow \boxed{t = \frac{\ln(5)}{\ln(1.03)} \approx 54.45}$$

19. $3(1.04)^{3t} = 8$

$$(1.04)^{3t} = \frac{8}{3}$$
$$\Rightarrow \ln\left((1.04)^t\right) = \ln\left(\frac{8}{3}\right)$$
$$\Rightarrow t\ln(1.04) = \ln\left(\frac{8}{3}\right)$$
$$\therefore \boxed{t = \frac{\ln\left(\frac{8}{3}\right)}{\ln(1.04)} \approx 25.008}$$

21. $50e^{-0.12t} = 10$

$$e^{-0.12t} = \frac{10}{50} = \frac{1}{5}$$
$$\Rightarrow \ln(e^{-0.12t}) = \ln\left(\frac{1}{5}\right) = -\ln(5)$$
$$\Rightarrow -0.12t = -\ln(5)$$
$$\therefore \boxed{t = \frac{\ln(5)}{0.12} \approx 13.412}$$

23. $10 - 8\left(\frac{1}{2}\right)^x = 5$

$$8\left(\frac{1}{2}\right)^x = 10 - 5 = 5$$

$$\Rightarrow \left(\frac{1}{2}\right)^x = \frac{5}{8}$$

$$\Rightarrow \ln\left(\left(\frac{1}{2}\right)^x\right) = \ln\left(\frac{5}{8}\right)$$

$$\Rightarrow x\ln\left(\frac{1}{2}\right) = \ln\left(\frac{5}{8}\right)$$

$$\Rightarrow -x\ln(2) = \ln\left(\frac{5}{8}\right)$$

$$\therefore \boxed{x = \frac{-\ln\left(\frac{5}{8}\right)}{\ln(2)} \approx 0.678}$$

25. The population of Kenya was 39.8 million in 2009 and has been growing by about 2.6% each year. If this trend continues, when will the population exceed 45 million?

Let $P(t)$ = the population in millions of Kenya t years after 2009.

Then, $P(t) = 39.8e^{0.026t}$

We must solve the equation $P(t) = 45$ for t.

$$39.8e^{0.026t} = 45$$

$$\Rightarrow e^{0.026t} = \frac{45}{39.8}$$

$$\Rightarrow \ln(e^{0.026t}) = \ln\left(\frac{45}{39.8}\right)$$

$$\Rightarrow 0.026t = \ln\left(\frac{45}{39.8}\right)$$

$$\therefore \boxed{t = \frac{\ln\left(\frac{45}{39.8}\right)}{0.026} \approx 4.72 years}$$

The population will exceed 45 million by the year 2014.

27. If $1000 is invested in an account earning 3% compounded monthly, how long will it take the account to grow in value to $1500?

Let $A(t)$ = the amount in the account after t years.

$$A(t) = 1000\left(1 + \frac{.03}{12}\right)^{12t}$$

We must solve the equation $A(t) = 1500$.

$$1000\left(1+\frac{.03}{12}\right)^{12t}=1500$$

$$\Rightarrow\left(1+\frac{.03}{12}\right)^{12t}=\frac{1500}{1000}=\frac{3}{2}$$

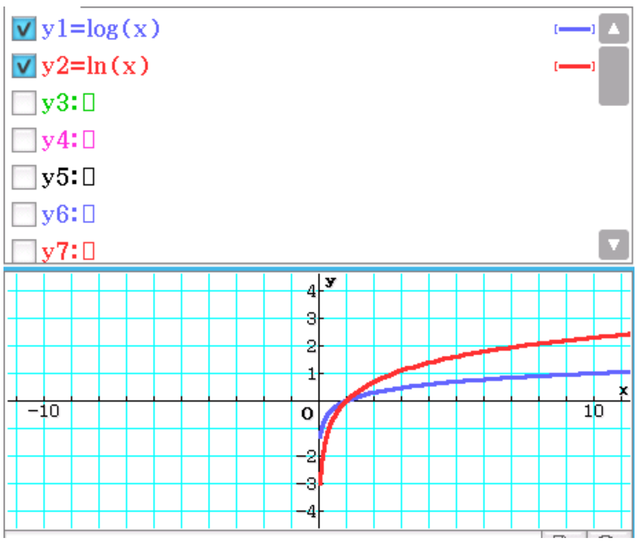
$$\Rightarrow\ln\left(\left(1+\frac{.03}{12}\right)^{12t}\right)=\ln\left(\frac{3}{2}\right)$$

$$\Rightarrow 12t\ln\left(1+\frac{.03}{12}\right)=\ln\left(\frac{3}{2}\right)$$

$$\therefore \boxed{t=\frac{\ln\left(\frac{3}{2}\right)}{12\ln\left(1+\frac{.03}{12}\right)}\approx 13.5 years}$$

29. Sketch a graph of: $f(x)=\log(x), g(x)=\ln(x)$

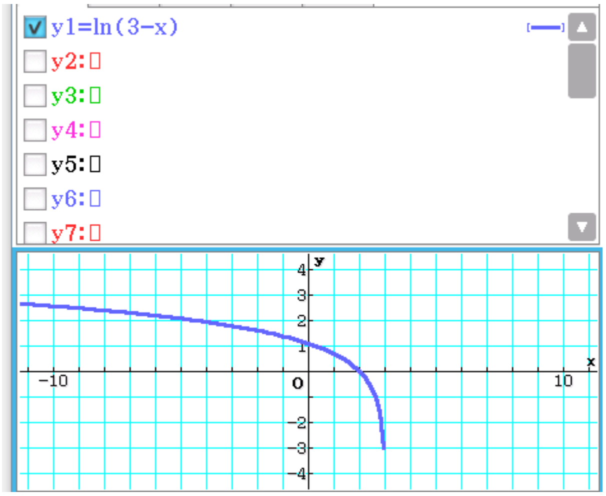
You should try this manually, but here is a calculator graph.



Find the domain of each function.

31. $f(x)=\ln(3-x)$

$f(x)$ is defined for $3-x>0 \Leftrightarrow x<3$.
Therefore the domain = $(-\infty, 3)=\{x\in\mathbb{R} | x<3\}$.
We can check this graphically.

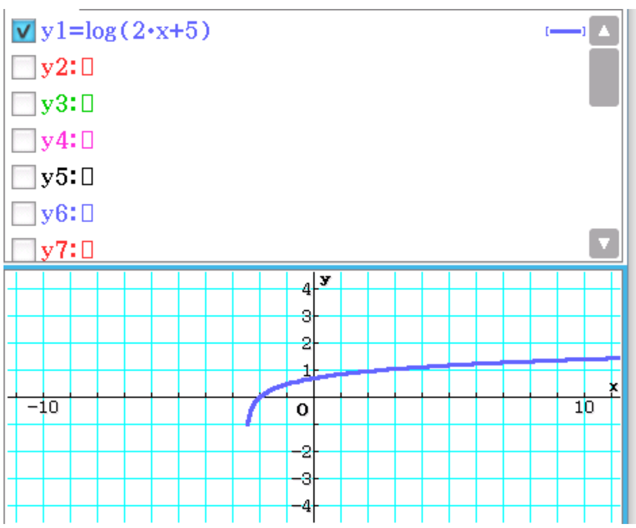


33. $f(x) = \log(2x+5)$

$f(x)$ is defined for $2x + 5 > 0 \Leftrightarrow x > -\frac{5}{2}$.

Therefore the domain $= \left(-\frac{5}{2}, \infty\right) = \left\{x \in \mathbb{R} | x > -\frac{5}{2}\right\}$.

We can check this graphically.



****
****

These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1*
by Shana Calaway, Dale Hoffman, David Lippman