1-5 Applied Calculus Solutions

Saturday, May 21, 2016 4:17 PM

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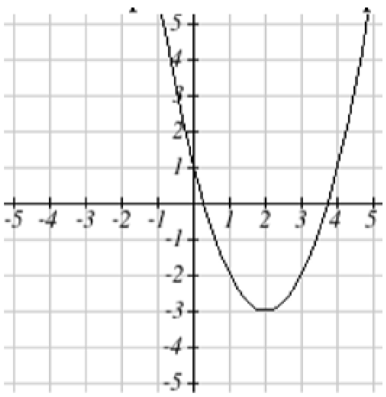
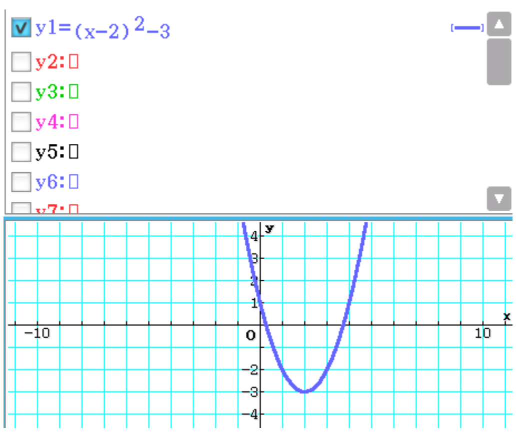
Write an equation for the quadratic function graphed.

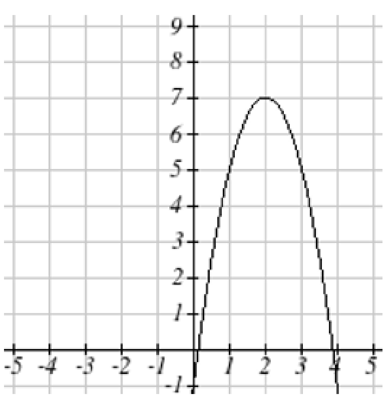
1. 

We begin with the basic quadratic function $y = x^2$. We than translate it to the right 2 units and then down 3 units.

$$y = (x-2)^2 - 3$$

You can check this by graphing it on your calculator.



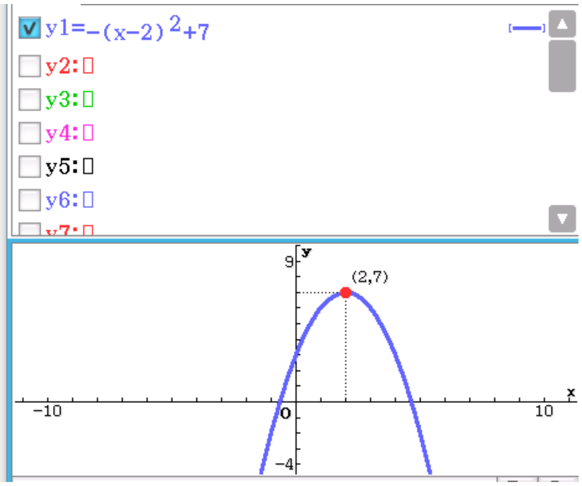
3. 

We begin with the basic quadratic equation $y = x^2$.
We shift it horizontally 2 units to the right.
We flip it vertically by multiplying by -1.

We then shift it vertically 7 units up.

$y = -(x-2)^2 + 7$

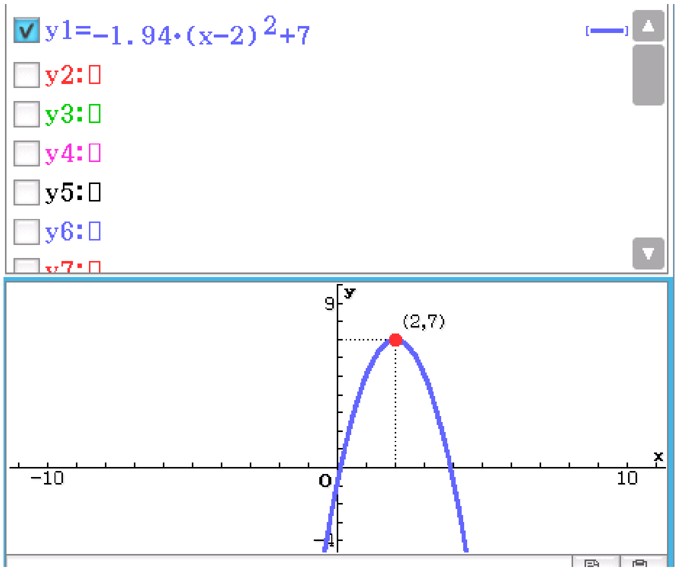
We check this by graphing the equation.



Hmmm. This parabola has the correct shape and the correct vertex, but it is too wide. We must multiply the quadratic term by some number greater than one. Here, we could guess and then check the result. Or, we could write the equation as $y = -c(x-2)^2 + 7$, set $y = 0$, set $y = 0.1$ (our estimate from the given graph), and solve for c.

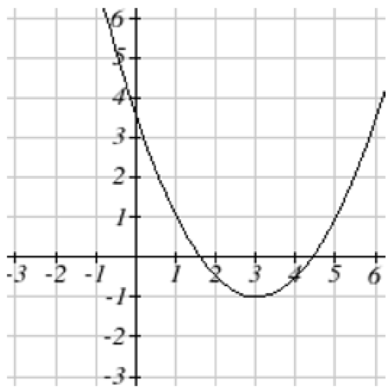
$$0 = -c(.1-2)^2 + 7$$
$$\Rightarrow 0 = -c(-1.9)^2 + 7$$
$$\Rightarrow 0 = -c(3.61) + 7$$
$$\Rightarrow c(3.61) = 7$$
$$\Rightarrow c = \frac{7}{3.61} \approx 1.94$$

Let's graph $y = -1.94(x-2)^2 + 7$.



This graph is close to the given graph.

5.



First, we transform the basic equation $y = x^2$
by shifting to the right 3 units and shifting down 1 unit.

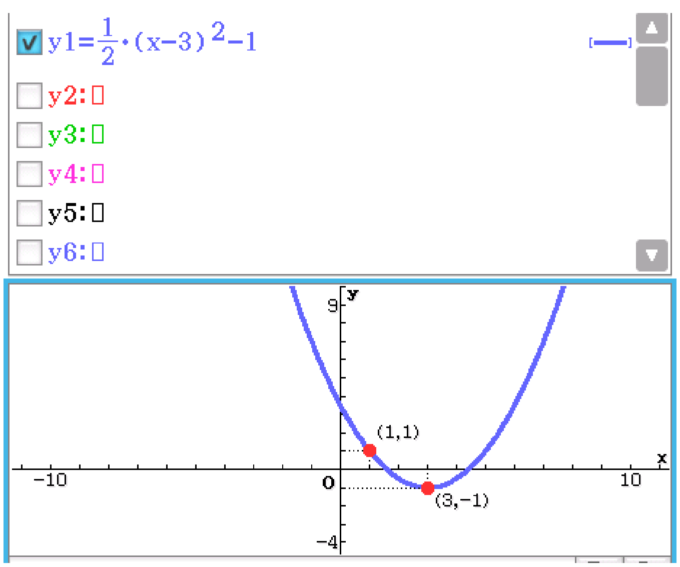
This gives $y = (x - 3)^2 - 1$.

To account for how wide the curve is, we must multiply the quadratic term by a number c such that the curve passes through a clearly identifiable point on the graph, say, $(1,1)$.

This gives the equation $1 = c(1 - 3)^2 - 1$, which we solve for c.

$\Rightarrow 1 = c(^{-2)2}-1$
$\Rightarrow 1 = c(4) - 1$
$\Rightarrow 4c = 1 + 1$
$\Rightarrow 4c = 2$
$\Rightarrow c = \frac{1}{2}$

The equation is $y = \frac{1}{2}(x - 3)^2 - 1$



This agrees with the given graph.

For each of the follow quadratic functions, find the vertical and horizontal intercepts.

7. $y(x) = 2x^2 + 10x + 12$

vertical intercept = y-intercept
Set $x = 0$ and solve for y.
$y(0) = 2(0^2) + 10(0) + 12 = 12$

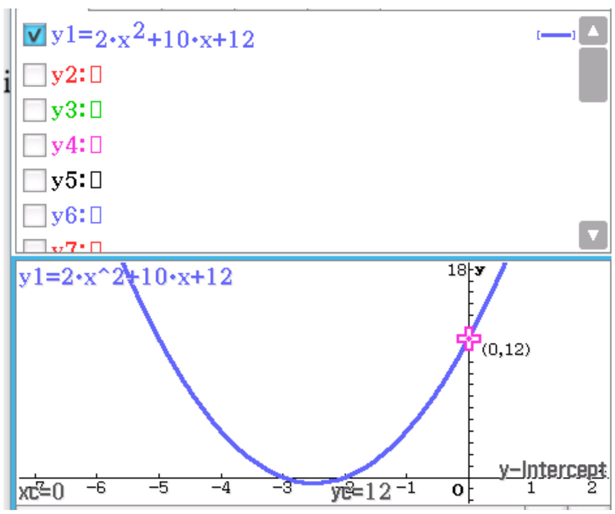
The vertical intercept is $y = 12$ or the point $(0,12)$.

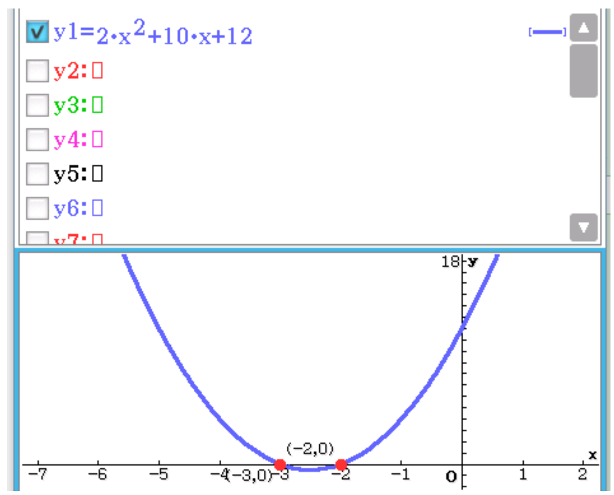
A horizontal intercept = x-intercept.
Set $y = 0$ and solve for x.

$$2x^2 + 10x + 12 = 0$$
$$\Rightarrow x^2 + 5x + 6 = 0$$
$$\Rightarrow (x+2)(x+3) = 0$$
$$\Rightarrow x = -2, -3$$

The x-intercepts are $x = -2, -3$ or the points $(-2,0)$ and $(-3,0)$.

We can check our results graphically.





11. $h(t) = -4t^2 + 6t - 1$

Vertical intercept = h-intercept
Set $t = 0$ and solve for h.
$h(0) = -4(0^2) + 6(0) - 1 = -1$
The vertical intercept is $h = -1$ or the point $(0, -1)$.

A horizontal intercept = t-intercept.
Set $h = 0$ and solve for t.

$$-4t^2 + 6t - 1 = 0$$

$$\Rightarrow t = \frac{-6 \pm \sqrt{36 - (4)(-4)(-1)}}{-8}$$

$$= \frac{-6 \pm \sqrt{36 - 16}}{-8} = \frac{-6 \pm \sqrt{20}}{-8}$$

$$= \frac{-6 \pm 2\sqrt{5}}{-8} = \frac{-3 \pm \sqrt{5}}{-4}$$

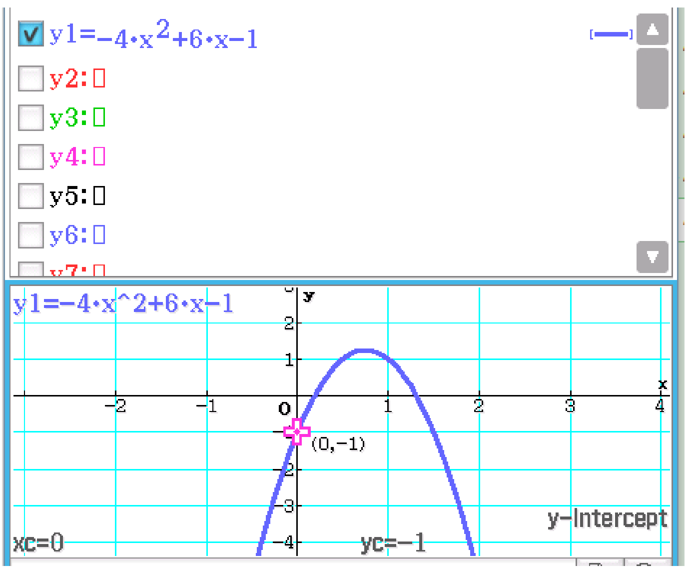
$$\Rightarrow t = \frac{3 \pm \sqrt{5}}{4}$$

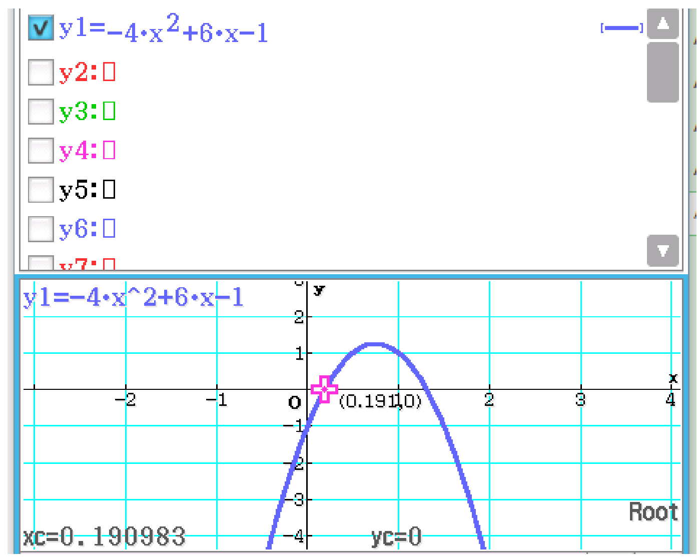
$$\frac{3-\sqrt{5}}{4}$$

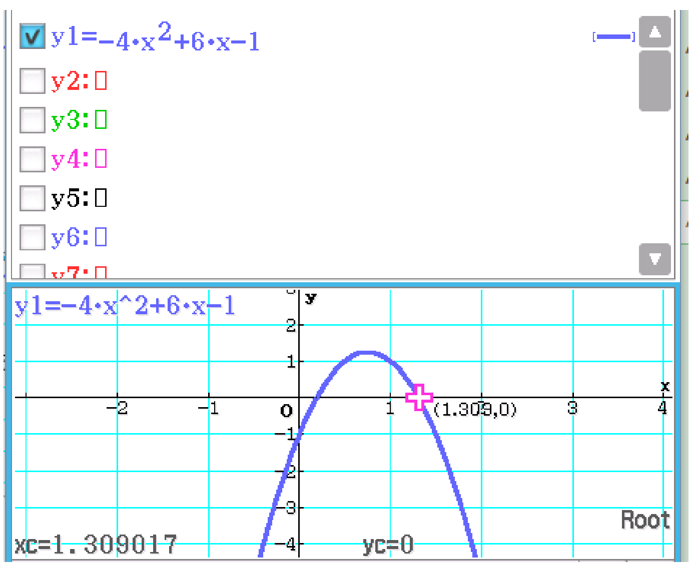
0.1909830056

$$\frac{3+\sqrt{5}}{4}$$

1.309016994







Our graphs agree with our calculations.

13. The height of a ball thrown in the air is given by $h(x) = -\frac{1}{12}x^2 + 6x + 3$, where x is the horizontal distance in feet from the point at which the ball is thrown.

a. How high is the ball when it was thrown?

b. What is the maximum height of the ball?

c. How far from the thrower does the ball strike the ground?

(13a) The ball was thrown when the distance is zero.

$$h(0) = -\frac{1}{12}(0^2) + 6(0) + 3 = 3feet$$

(13b) The maximum height of the ball is at the vertex of the parabola. We can find this by completing the square to rewrite the equation in the form $h(x) = a(x - p)^2 + k$, where the vertex is the point (p, k).

We do this by completing the square.

$$h(x) = \left(\frac{-1}{12}x^2 + 6x\right) + 3$$
$$= -\frac{1}{12}(x^2 - 72x) + 3$$
$$= -\frac{1}{12}(x^2 - 72x + 36^2 - 36^2) + 3$$
$$= -\frac{1}{12}(x^2 - 72x + 36^2) - \frac{1}{12}(-36^2) + 3$$
$$= -\frac{1}{12}(x - 36)^2 + \frac{(12)(3)(36)}{12} + 3$$
$$= -\frac{1}{12}(x - 36)^2 + 108 + 3$$
$$= -\frac{1}{12}(x - 36)^2 + 111$$

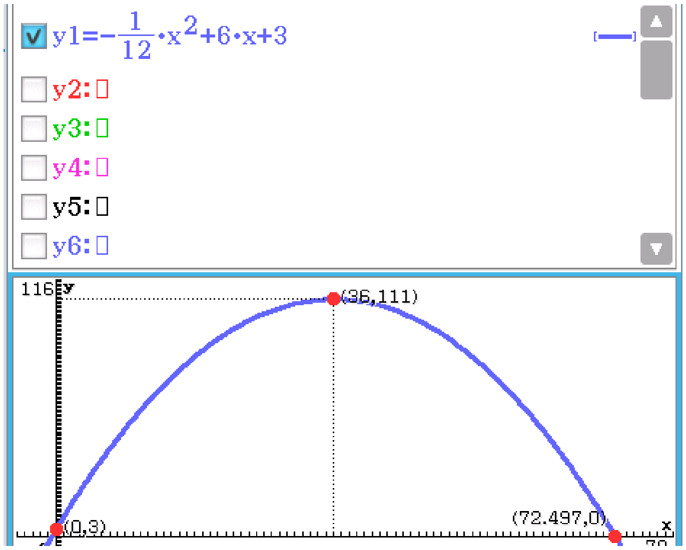
Therefore the vertex is $(36{,}111)$,
and so the maximum height of the ball is 111 feet.

(13c) The ball strikes the ground when $h(x) = 0$.

$$-\frac{1}{12}(x - 36)^2 + 111 = 0$$
$$\Rightarrow (x - 36)^2 - (12)(111) = 0$$
$$\Rightarrow (x - 36)^2 = 12(111)$$
$$\Rightarrow x - 36 = \pm\sqrt{(12)(111)}$$
$$\Rightarrow x = 36 \pm \sqrt{(4)(3)(3)(37)}$$
$$\Rightarrow x = 36 \pm 6\sqrt{37}$$

We cannot have a negative value for x,
so we choose $x = 36 + 6\sqrt{37} \approx 72.5feet$

We check these results graphically.



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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1* by Shana Calaway, Dale Hoffman, David Lippman