1-3 Applied Calculus Solutions

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1. A town's population has been growing linearly. In 2003, the population was 45,000, and the population has been growing by 1700 people each year. Write an equation, $P(t)$, for the population t years after 2003.

$P(t) = 45{,}000 + 1700t$

As a check, we can calculate $P(0) = 45{,}000 + 1700(0) = 45{,}000$.

Also, the function is of the general linear form: $y = mx + b$

3. Timmy goes to the fair with \$40. Each ride costs \$2. How much money will he have left after riding n rides?

Let $M(n) =$ the amount of money Timmy has left after n rides.

$M(n) = \$40 - \$(2n)$

5. A phone company charges for service according to the formula: $C(n) = 24 + 0.1n$, where n is the number of minutes talked, and $C(n)$ is the monthly charge, in dollars. Find and interpret the rate of change and initial value.

The rate of change is 0.1.
The cost increases at a rate of 10 cents per minute talked.

The initial value is $C(0) = 24 + 0.1(0) = \$24$

Given each set of information, find a linear equation satisfying the conditions, if possible

7. $f(-5) = -4$, and $f(5) = 2$

Let $f(x) = mx + b$

$\Rightarrow m(-5) + b = -4$
$m(5) + b = 2$

$\Rightarrow 2b = -2$

$\Rightarrow b = -1$

$\Rightarrow 5m - 1 = 2$

$\Rightarrow 5m = 3$

$\Rightarrow m = \frac{3}{5}$

$$\boxed{\therefore f(x) = \frac{3}{5}x - 1}$$

9. Passes through (2, 4) and (4, 10)

slope of line = $\frac{10-4}{4-2} = \frac{6}{2} = 3$

$y = 3x + b$

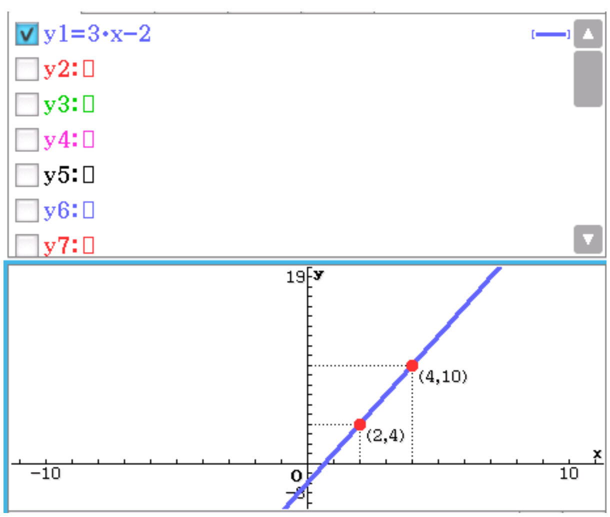
$4 = (3)(2) + b$

$b = 4 - 6$

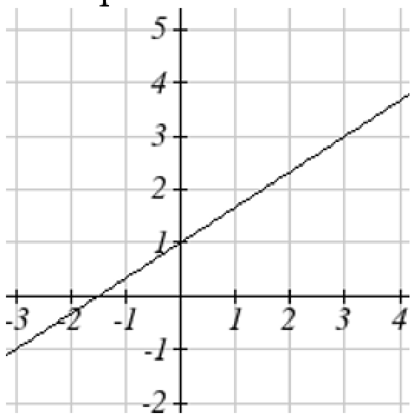
$b = -2$

Equation of line is $y = 3x - 2$

As a check, we can graph this equation and use trace to verify that the given points lie on the line.



15. Find an equation for the function graphed



From the graph, we can estimate that two point on the line are $(0,1)$ and $(3,3)$.

The slope of the line is $\frac{3-1}{3-0} = \frac{2}{3}$.

$$y = \left(\frac{2}{3}\right)x + b$$

$$1 = \left(\frac{2}{3}\right)(0) + b$$

$$b = 1$$

The equation of the line is $y = \frac{2x}{3} + 1$

17. A clothing business finds there is a linear relationship between the number of shirts, n, it can sell and the price, p, it can charge per shirt. In particular, historical data shows that 1000 shirts can be sold at a price of \$30, while 3000 shirts can be sold at a price of \$22. Find a linear equation in the form $p = mn + b$ that gives the price p they can charge for n shirts.

The given information implies that $p(1000) = 30$ and $p(3000) = 22$.

$\Rightarrow$ two points on the line given by $p = mn + b$ are $(30{,}1000)$ and $(22{,}3000)$.

$\Rightarrow$ slope of line = $\frac{3000-1000}{22-30} = \frac{2000}{-8} = -250$

$\Rightarrow p = -250n + b$

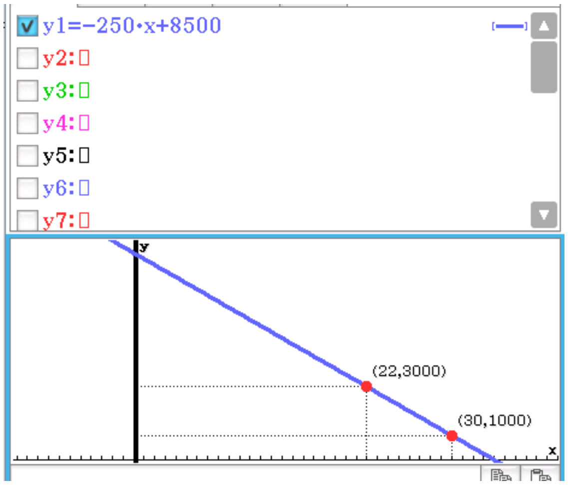
$\Rightarrow 1000 = -250(30) + b$

$\Rightarrow b = 1000 + 7500$

$\Rightarrow b = 8500$

The equation is $p = -250n + 8500$.

We check our work by graphing the equation and using trace to verify that the calculated points lie on the line.



29. Find the point at which the line $f(x) = -2x - 1$ intersects the line $g(x) = -x$

We can solve this problem algebraically and graphically.

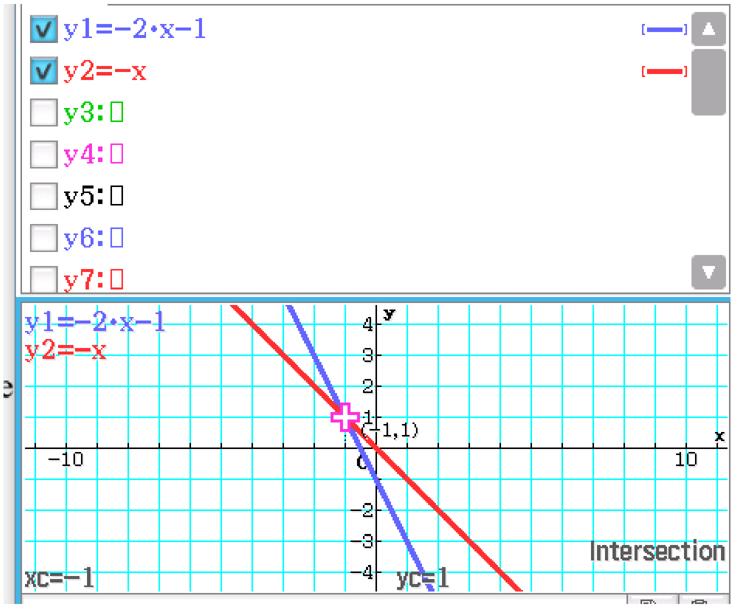
For algebra, solve the equation $-2x - 1 = -x$.

$-x - 1 = 0$

$x = -1$

$\Rightarrow g(-1) = -(-1) = 1$

$\therefore$ the intersection point is $(-1,1)$



Our answers agree!

31. A car rental company offers two plans for renting a car.
Plan A: 30 dollars per day and 18 cents per mile
Plan B: 50 dollars per day with free unlimited mileage
How many miles would you need to drive for plan B to save you money?

Let $A_1(m)$ = the cost with Plan A for driving m miles in one day.
Let $B_1(m)$ = the cost with Plan B for driving m miles in one day.

We must find m so that $B_1(m) < A_1(m)$.

$A_1(m) = 30 + .18m$

$B_1(m) = 50$

We must solve the inequality $50 < 30 + .18m$

$20 < .18m$

$\frac{20}{.18} < m$

$111\frac{1}{9} < m$

Thus, for one day, you would save money with Plan B if you drive more than $111\frac{1}{9}$ miles.

What happens if you drive 2 days? Will the mileage change or not?

Let $A_2(m)$ = the cost with Plan A for driving m miles in one day.
Let $B_2(m)$ = the cost with Plan B for driving m miles in one day.

We must find m so that $B_2(m) < A_2(m)$.

$A_2(m) = 60 + .18m$

$B_2(m) = 100$

$100 < 60 + .18m$

$$40 < .18m$$
$$\frac{40}{.18} < m$$
$$22\frac{2}{9} < m$$

Thus, for two days, you would save money with Plan B if you drive more than $22\frac{2}{9}$ miles.

Therefore, how you answer this question depends on how many travel days you plan.

33. The Federal Helium Reserve held about 16 billion cubic feet of helium in 2010, and is being depleted by about 2.1 billion cubic feet each year.

    a. Give a linear equation for the remaining federal helium reserves, *R*, in terms of *t*, the number of years since 2010.
    b. In 2015, what will the helium reserves be?
    c. If the rate of depletion doesn't change, when will the Federal Helium Reserve be depleted?

(33a) $R(t) = 16 - 2.1t$, where $t =$ the number of years after 2010 and $R(t)$ is measured in billions of cubic feet.

(33b) $R(5) = 16 - 2.1(5) = 16 - 10.5 = 5.5$ billion cubic feet

(33c) When will $R(t) = 0$?

$$16 - 2.1t = 0$$
$$2.1t = 16$$
$$t = \frac{16}{2.1} \approx 7.6 \text{ years}$$

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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1* by Shana Calaway, Dale Hoffman, David Lippman