04-01-25 MDE 61

3.3 Real Zeros of Polynomials
3.3.3: Exercises
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Long division

$(6x^4 - 3x^2 + 2x + 7) \div (x^2 + 5)$

$$
\begin{array}{r}
6x^2 \qquad\qquad -33 + \frac{2x+172}{x^2+5} \\
x^2+5 \overline{\big)\, 6x^4 + 0x^3 - 3x^2 + 2x + 7} \\
\underline{6x^4 + 0x^3 \;\; 30x^2} \qquad\quad \\
-33x^2 + 2x + 7 \\
\underline{-33x^2 \qquad -165} \\
2x + 172
\end{array}
$$

$(6x^4 - 3x^2 + 2x + 7) \div (x + 5)$

$$
\begin{array}{r|rrrrr}
-5 & 6 & 0 & -3 & 2 & 7 \\
 & & -30 & 150 & -735 & 3665 \\
\hline
 & 6 & -30 & 147 & -733 & (3672)
\end{array}
$$

$6x^3 - 30x^2 + 147x - 733 + \frac{3672}{x+5}$

since remainder $\neq 0$

$x + 5$ is not a factor of the poly

and $x = -5$ is not a zero of the poly

Quiz 4

Theorem 3.9. Rational Zeros Theorem: Suppose $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ is a polynomial of degree n with $n \geq 1$, and $a_0, a_1, \ldots a_n$ are integers. If r is a rational zero of f, then r is of the form $\pm\frac{p}{q}$, where p is a factor of the constant term a_0, and q is a factor of the leading coefficient a_n.

Theorem 3.10. Descartes' Rule of Signs: Suppose $f(x)$ is the formula for a polynomial function written with descending powers of x.

- If P denotes the number of variations of sign in the formula for $f(x)$, then the number of positive real zeros (counting multiplicity) is one of the numbers $\{P, P-2, P-4, \ldots\}$.
- If N denotes the number of variations of sign in the formula for $f(-x)$, then the number of negative real zeros (counting multiplicity) is one of the numbers $\{N, N-2, N-4, \ldots\}$.

Let $f(x) = \underbrace{6x^4 - 3x^2}_{?} \underbrace{+ 2x}_{?} \underbrace{+ 7}_{?}$

Use theorem 3.9 to find any possible rational zeros of f(x). Use Theorem 3.10 to predict the number of positive and negative real zeros of f(x). Then, find the real zeros of f(x) graphically and compare the results with your predictions.

$a \mid b$

"a divides into b"

"a is a factor of b"

$a_0 = 7$

$a_4 = 6$

$r = \frac{p}{q}$

$p \mid a_0 \Rightarrow p \mid 7 \Rightarrow p = \pm 1, \pm 7$

$q \mid a_4 \Rightarrow q \mid 6 \Rightarrow q = \pm 1, \pm 2, \pm 3, \pm 6$

$$\boxed{\frac{p}{q} = \pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm 7, \pm\frac{7}{2}, \pm\frac{7}{3}, \pm\frac{7}{6} \quad \text{possible rational zeros}}$$

Let $f(x) = \underbrace{6x^4 - 3x^2}_{1} \underbrace{+ 2x}_{1} \underbrace{+ 7}_{0}$

Let $f(x) = 6x^4 - 3x^2 + 2x + 7$

1 1 0

2 sign changes

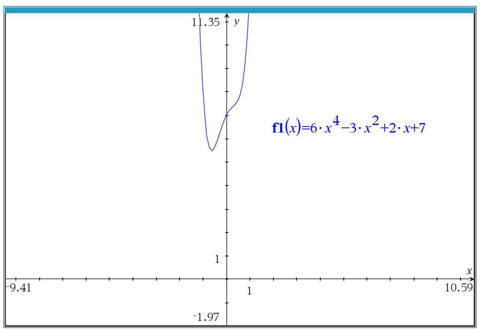
$\Rightarrow$ there are 2 or $2-2=0$ positive real zeros

$$f(-x) = 6(-x)^4 - 3(-x)^2 + 2(-x) + 7$$
$$= 6x^4 - 3x^2 - 2x + 7$$

1 0 1

2 sign changes

$\therefore$ 2 or 0 negative real zeros



From the graph,
there are no real zeros

This result agrees with the predictions of the theorems.

factoring

Factor 120 into powers of prime factors

120

2 60

5 12

2 6

2 3

$$\boxed{120 = 2^3 \cdot 3 \cdot 5}$$

Factor $x^2 + 6x^3$

$$= x(x + 6x^2)$$
$$= (x)(x)(1 + 6x)$$
$$= x^2(1 + 6x)$$

Factor $36x^4 - 12x^3 + 2x^2$

Factor $36x^7 - 12x^3 + 2x^2$

$= 2x^2\underbrace{(18x^2 - 6x + 1)}$

? $(2x-1)(9x-1)$ No

? $(6x-1)(3x-1)$ No

? $(18x-1)(x-1)$ No

Rational zeros

$p \mid 0 \Rightarrow p = 0$

$q \mid 36$

$\boxed{\frac{p}{q} = 0}$

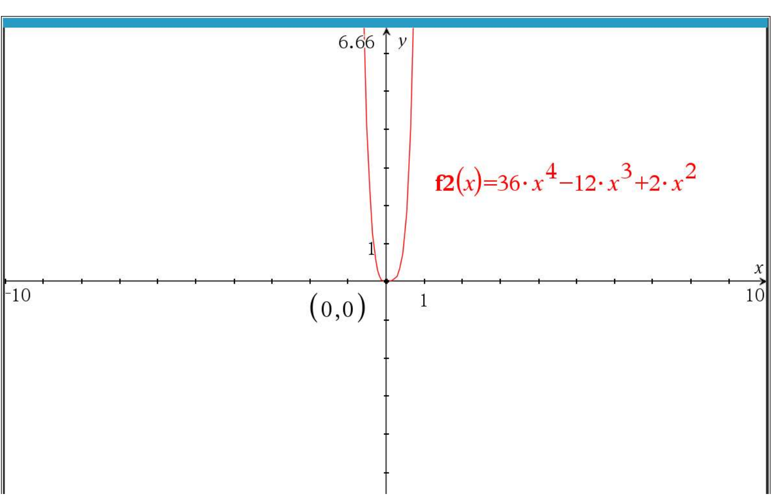
$f(x) = \underbrace{36x^4 - 12x^3}_{1} \underbrace{{} + 2x^2}_{1}$

2 or 0 pos. real zero

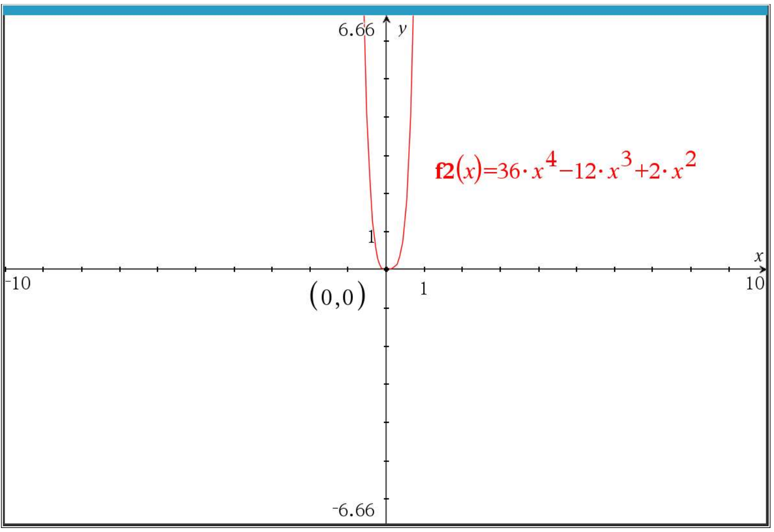
$f(-x) = 36(-x)^4 - 12(-x)^3 + 2(-x)^2$

$= \underbrace{36x^4 + 12x^3}_{0} \underbrace{{} + 2x^2}_{0}$

0 neg real zero



real zero is $\boxed{x = 0}$



real zero

is $x=0$