03-27-25 MDE 61

3.2 The Factor Theorem and the Remainder Theorem
3.2.1 Exercises
page 265: 1, 3, 9, 21, 35, 42

3.2

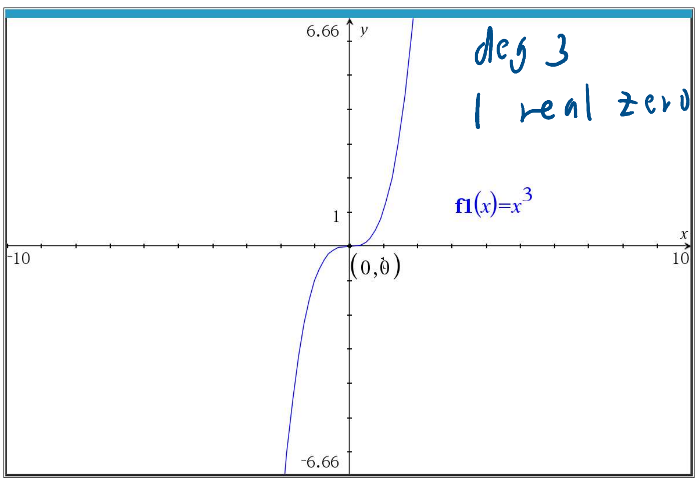
Memorize

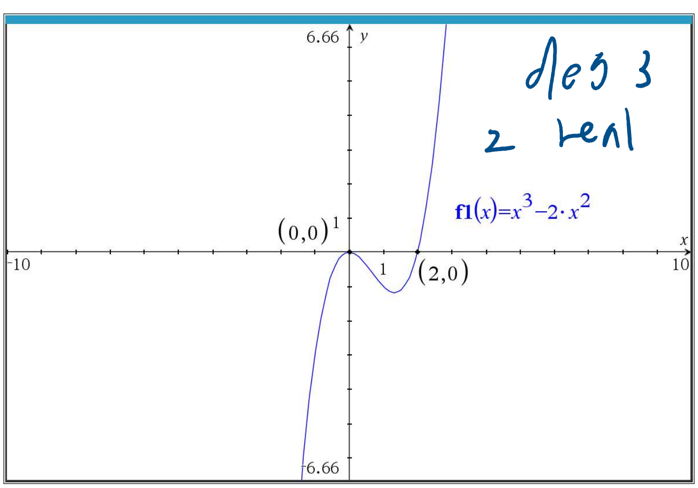
Theorem 3.7. Suppose f is a polynomial of degree $n \geq 1$. Then f has at most n real zeros, counting multiplicities.

Memorize

Definition 1.9. The **zeros** of a function f are the solutions to the equation $f(x) = 0$. In other words, x is a zero of f if and only if $(x, 0)$ is an x-intercept of the graph of $y = f(x)$.

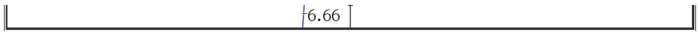
Definition 3.3. Suppose f is a polynomial function and m is a natural number. If $(x - c)^m$ is a factor of $f(x)$ but $(x - c)^{m+1}$ is not, then we say $x = c$ is a zero of **multiplicity** m.





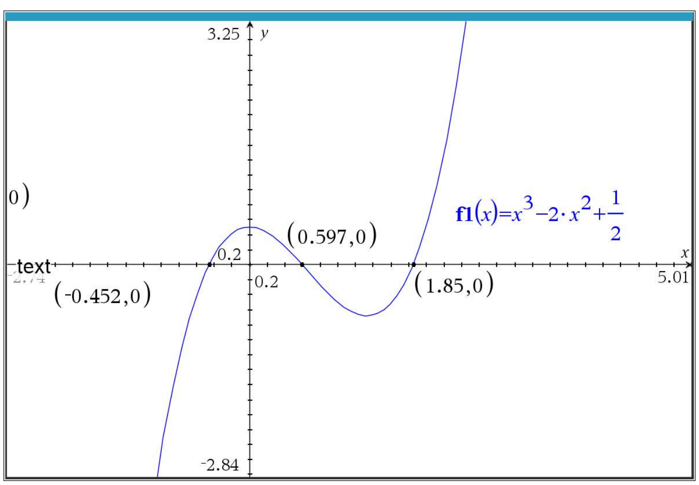
zero/

$f(x) = x^3 - 2x^2$



$$f(x) = x^3 - 2x^2$$
$$= x^2(x-2) = (x-0)^2(x-2)$$

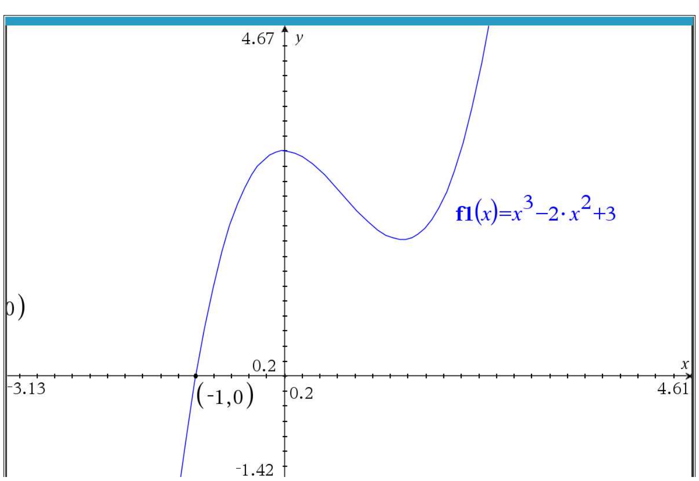
zero	multiplicity
$x=0$	2
$x=2$	1
	3



deg = 3

3 distinct real zeros each with multiplicity 1

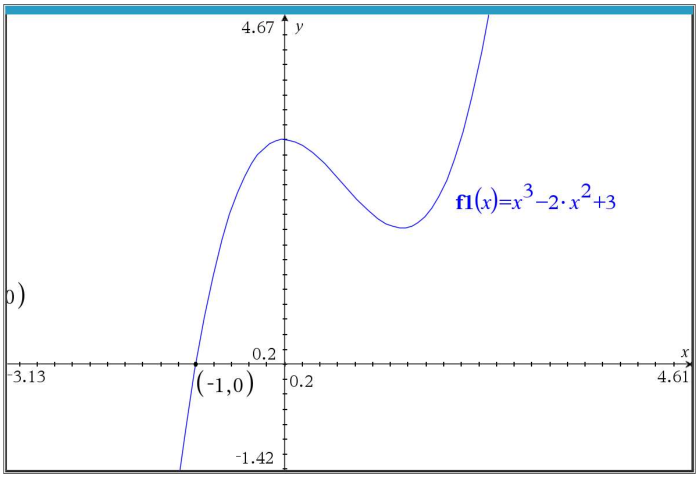
solve$(x^3=0,x)$ $x=0$



deg = 3

1 real zero $x=-1$ mult. 1

2 complex solutions



deg = 3

1 real zero $x=-1$
mult. 1

2 complex solutions

solve$(x^3-2\cdot x^2+3=0,x)$	$x=-1$
cSolve$(x^3-2\cdot x^2+3=0,x)$	$x=\frac{3}{2}+\frac{\sqrt{3}}{2}\cdot \boldsymbol{i}$ or $x=\frac{3}{2}-\frac{\sqrt{3}}{2}\cdot \boldsymbol{i}$ or $x=-1$

$\boxed{i^2=-1}$ memorize

Memorize

Connections Between Zeros, Factors and Graphs of Polynomial Functions

Suppose p is a polynomial function of degree $n \geq 1$. The following statements are equivalent:

- The real number c is a zero of p
- $p(c) = 0$
- $x = c$ is a solution to the polynomial equation $p(x) = 0$
- $(x - c)$ is a factor of $p(x)$
- The point $(c, 0)$ is an x-intercept of the graph of $y = p(x)$

3.2:

In Exercises 31 - 40, you are given a polynomial and one of its zeros. Use the techniques in this section to find the rest of the real zeros and factor the polynomial.

32. $x^3 - 24x^2 + 192x - 512, \quad c = 8$

Let $p(x) = x^3 - 24x^2 + 192x - 512$

8^3-24*8^2+192*8-512=0

$$\begin{array}{r|l} & x^2 \quad -16x \quad +64 \\ \hline x-8 & x^3 \quad -24x^2 \quad +192x \quad -512 \\ & x^3 \quad -8x^2 \\ \hline \end{array}$$

$x^3 - 8x^2$

$-16x^2 + 192x$

$-16x^2 + 128x$

$64x - 512$

$64x - 512$

0

$p(8) = 0$

$p(x) = (x-8)(x^2 - 16x + 64)$

$p(x) = 0$ if $\boxed{x = 8}$ or $x^2 - 16x + 64 = 0$

mult. 1

$(x-8)^2 = 0$

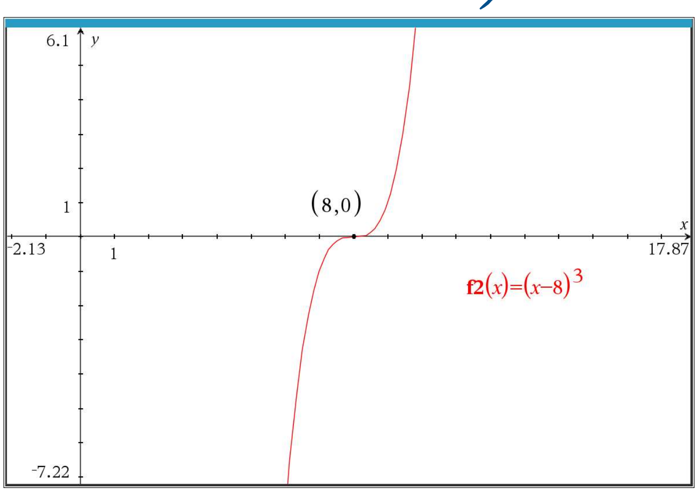
$\boxed{x = 8 \text{ mult } 2}$

$x = 8$ mult 3

$p(x) = (x-8)^3$

3 is odd

$\Rightarrow$ graph crosses x-axis at $x = 8$



Synthetic division: only applies when divisor is a linear function

32. $x^3 - 24x^2 + 192x - 512, \quad c = 8$

linear function

32. $x^3 - 24x^2 + 192x - 512, \quad c = 8$

$(x^3 - 24x^2 + 192x - 512) \div x - 8$

multiply

$$\begin{array}{r|rrrr} 8 & 1 & -24 & 192 & -512 \\ & & 8 & -128 & 512 \\ \hline & 1 & -16 & 64 & (0) \end{array}$$

add

remainder

$x^2 - 16x + 64$

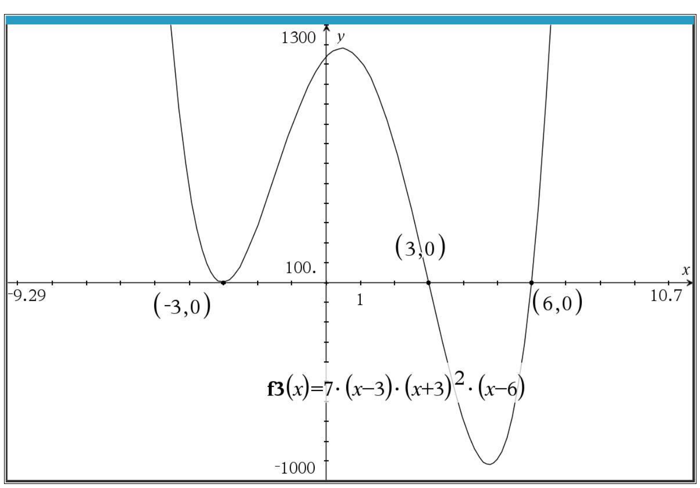
3.1

In Exercises 41 - 45, create a polynomial p which has the desired characteristics. You may leave the polynomial in factored form.

43.
- The solutions to $p(x) = 0$ are $x = \pm 3$ and $x = 6$
- The leading term of $p(x)$ is $7x^4$
- The point $(-3, 0)$ is a local minimum on the graph of $y = p(x)$.

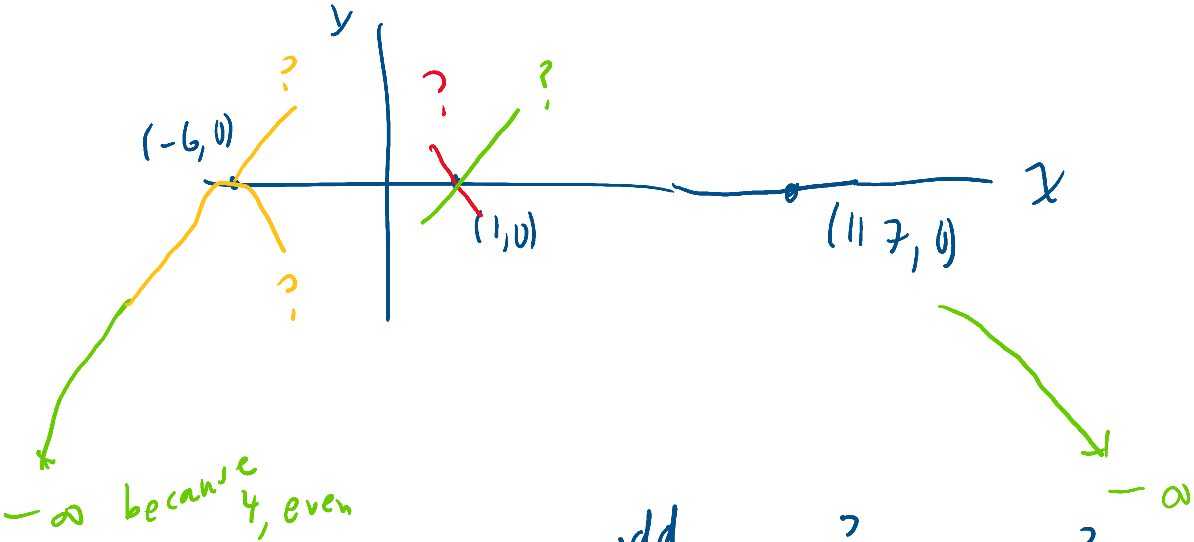
$p(x) = 7(x-3)(x-6)(x+3)^2 \cancel{(x)}$

$= 7x^4 + \ldots$



45.

- p is degree 4.
- as $x \to \infty$, $p(x) \to -\infty$
- p has exactly three x-intercepts: $(-6, 0)$, $(1, 0)$ and $(117, 0)$
- The graph of $y = p(x)$ crosses through the x-axis at $(1, 0)$.



$$p(x) = a(x-1)^{odd}(x+6)^{?}(x-117)^{?}$$

$$x \to \infty \quad p(x) = a(+)(+)(+) \Rightarrow -\infty$$

$$\Rightarrow a < 0$$

$$x \to \infty \quad p(x) = \underbrace{-(-)^{odd\ m}}_{-}(-)^{n}(-)^{q} \to -\infty$$

Finish at home

$$m + n + q = 4$$

m: odd