03-25-25 MDE 61

3.1 Graphs of Polynomials

3.1.1 Exercises

page 246: 3, 7, 13, 21, 27

3.1:

> **Theorem 3.1. The Intermediate Value Theorem (Zero Version):** Suppose f is a continuous function on an interval containing $x = a$ and $x = b$ with $a < b$. If $f(a)$ and $f(b)$ have different signs, then f has at least one zero between $x = a$ and $x = b$; that is, for at least one real number c such that $a < c < b$, we have $f(c) = 0$.

$$f(x) = (x+4)(x-3)(x-7)$$

$$f(x) = (x^2+x-12)(x-7)$$

$$f(x) = x^3 - 7x^2 + x^2 - 7x - 12x + 84$$

$$\boxed{f(x) = x^3 - 6x^2 - 19x + 84}$$

Use the intermediate value theorem to show that there is at least one zero in the interval [4,9].

$$a = 4, \quad b = 9$$

$$f(4) = (4+4)(4-3)(4-7)$$

$$= (+)(+)(-) < 0$$

$$f(9) = (9+3)(9-3(9-7)$$

$$f(9) = (9+3)(9-3)(9-7)$$
$$= (+)(+)(+) > 0$$

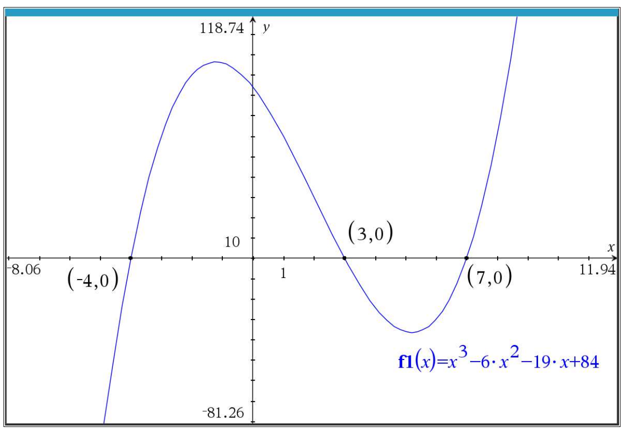
Therefore, by the intermediate value theorem (IVT), there exists at least one value c with 4 < c < 9 and f (c) = 0.

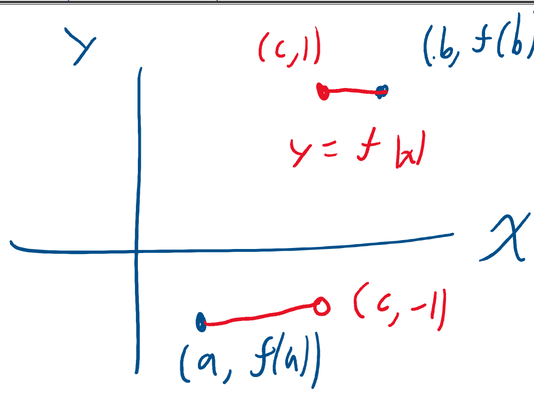
Without factoring, we can still do this problem

$$f(4) = 4^3 - 6(4^2) - 19(4) + 84 = -24 < 0$$

4^3-6*4^2-19*4+84=-24

f(9) = 9^3-6*4^2-19*4+84=641 >0





This example does not violate the IVT, because the function is not continuous.
If all of the conditions of a theorem are not satisfied, then the conclusion of the theorem is not guaranteed.

solve algebraically $|x+4| = 9$

$$x + 4 = \pm 9$$

$x + 4 = -9$ or $x + 4 = 9$

$$\boxed{x=-13,\ 5}$$

check $|-13+4| \stackrel{?}{=} 9$

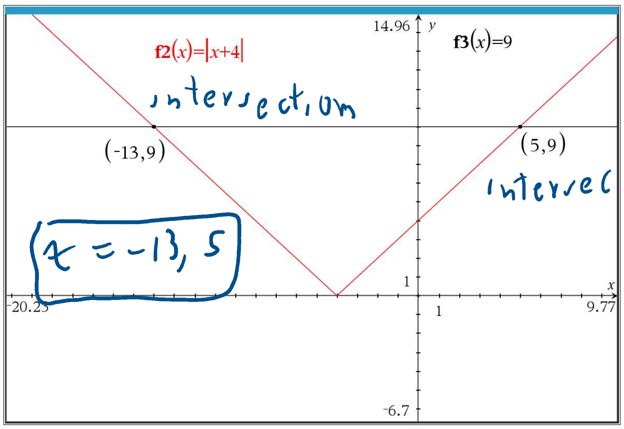
$|-9| \stackrel{?}{=} 9$

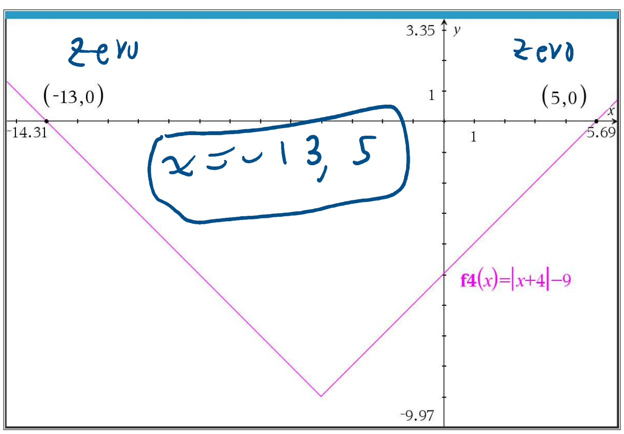
$9 = 9$ ✓

$|5+4| \stackrel{?}{=} 9$

$|9| \stackrel{?}{=} 9$

$9 = 9$ ✓





2.2

In Exercises 1 - 15, solve the equation.

6. $|7x-1|+2=0$

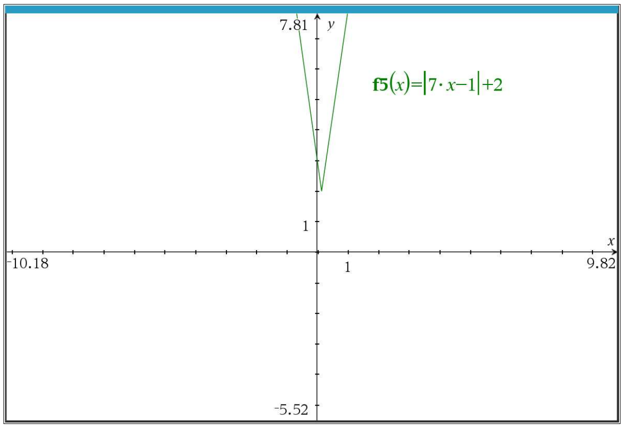
$\Rightarrow |7x-1| = -2 < 0$

$\therefore$ no solution

$x = \frac{1}{7}$

no solution ✓

$x = -\frac{1}{7}, \frac{3}{7}$



From the graph, there is no x-intercept.
Therefore, there is no solution.

Find the slope-intercept equation of the line passing through the points (3,4) and (2,5).

slope-intercept form $y = mx + b$

m = slope, b = y-intercept

$$m = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{5-4}{2-3} = \frac{1}{-1} = -1$$

$$\boxed{m = -1}$$

point-slope form

$(y - y_0) = m(x - x_0)$, m = slope, (x_0, y_0) = fixed point

$y - 4 = -(x - 3)$

convert to slope-intercept

$y = -x + 3 + 4$

$$\boxed{y = -x + 7}$$

another approach

$\boxed{m = -1}$

another approach

From above we have $\boxed{m = -1}$

same

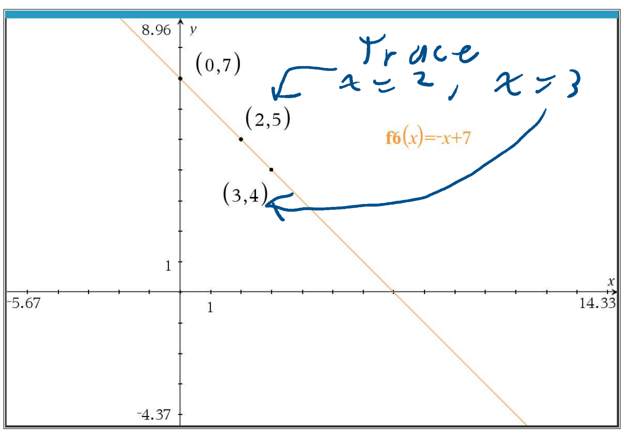
$$y = mx + b$$

$$\boxed{y = -x + b}$$

$$4 = -3 + b$$

$$\boxed{b = 7}$$

$$\boxed{y = -x + 7}$$



Using Trace, we verified that our calculated line contains the two given points.