Review for exam 2

Exam 2
Thursday, 03/13/25 (changed from Wednesday)
1.6-1.7,2.1 -2.4

Definition: a theorem is a proven mathematical statement.

We prove a theorem or proposition by deducing it from axioms, definitions, and previously proved theorems, by agreed upon methods of logic.

Given the definition of a rectangle as a 4-sided polygon with 4 right angles and the definition of a square as a 4-sided polygon with equal sides and 4 right angles.
Prove that a square is a rectangle.

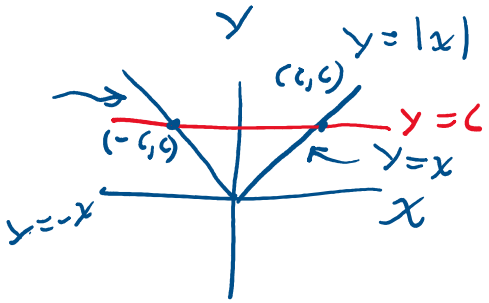
By definition of a square, a square has 4 sides and 4 right angles. Thus, the square satisfies the definition of a rectangle.
Q.E.D Latin "quod erat demonstradum" = "that which was to be proved.

2.2

2.2.1 Exercises

In Exercises 1 - 15, solve the equation.

11. $4 - |x| = 2x + 1$



solve $|x| = c$

$x = \pm c$

$-|x| = 2x - 3$

$|x| = 3 - 2x$ or $|x| = 2x + 1$

$x = 3 - 2x$ or $x = -3 + 2x$

$3x = 3$

$x = 1$

$-x = -3$

or $x = 3$

reject extraneous solution

Check

$4 - |1| \stackrel{?}{=} 2(1) + 1$

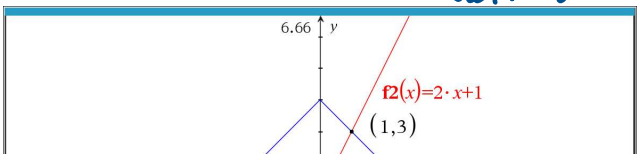
$4 - 1 \stackrel{?}{=} 2 + 1$

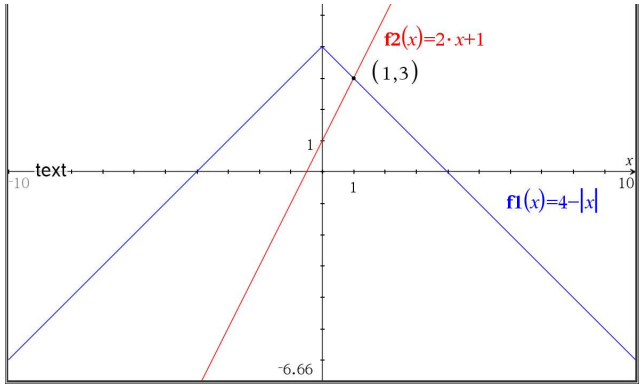
$3 = 3$ ✓

$4 - |3| \stackrel{?}{=} 2(3) + 1$

$4 - 3 \stackrel{?}{=} 6 + 1$

$1 \neq 7$





from graph $x = 1$

$1 - 3 = 6 + 1$

$1 \neq 7$

$\therefore x = 3$ is an extraneous solution

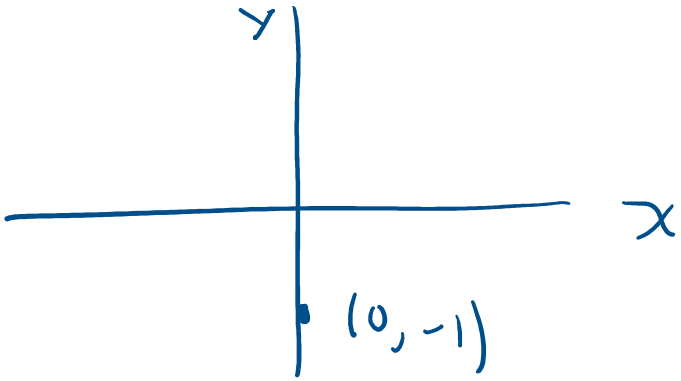
2.3.1 EXERCISES

In Exercises 1 - 9, graph the quadratic function. Find the x- and y-intercepts of each graph, if any exist. If it is given in general form, convert it into standard form; if it is given in standard form, convert it into general form. Find the domain and range of the function and list the intervals on which the function is increasing or decreasing. Identify the vertex and the axis of symmetry and determine whether the vertex yields a relative and absolute maximum or minimum.

9.[10] $f(x) = x^2 - \frac{1}{100}x - 1$

y-intercept

$f(0) = 0^2 - \left(\frac{1}{100}\right)(0) - 1 = -1$



x-intercept

$x^2 - \left(\frac{1}{100}\right)x - 1 = 0$

$x = \dfrac{\frac{1}{100} \pm \sqrt{\frac{1}{10000} + 4}}{2}$

$x = \dfrac{\frac{1}{100} \pm \sqrt{\frac{40001}{10000}}}{2}$

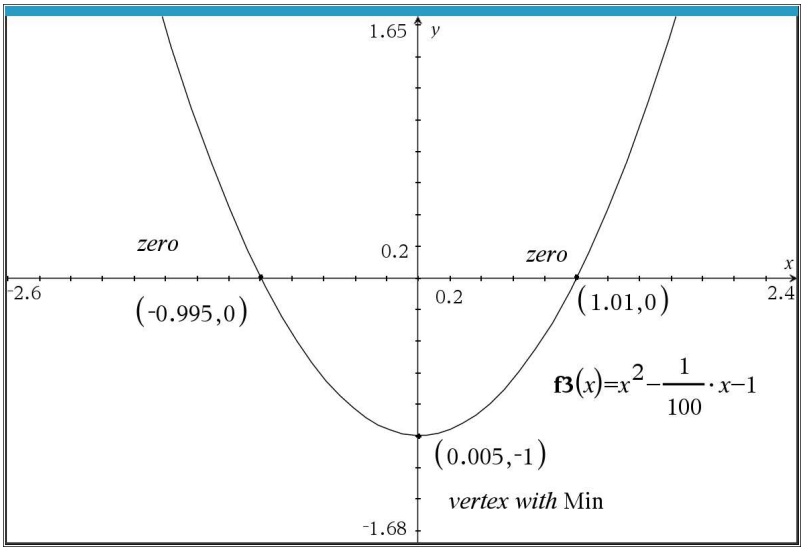
$x = \frac{1}{200} \pm \frac{\sqrt{40001}}{200}$

$x = \dfrac{1 \pm \sqrt{40001}}{200}$

200

(1-sqrt(40001))/200=-0.995012499921876

(1+sqrt(40001))/200=1.005012499921876



9.[10] $f(x) = x^2 - \frac{1}{100}x - 1$

Convert to vertex form by completing the square

$f(x) = \left(x^2 - \frac{x}{100}\right) - 1$

$\left(\frac{1}{2}\right)\left(-\frac{1}{100}\right) = -\frac{1}{200}$

$\left(-\frac{1}{200}\right)^2 = \frac{1}{40000}$

$f(x) = \left(x^2 - \frac{x}{100} + \frac{1}{40000} - \frac{1}{40000}\right) - 1$

$f(x) = \left(x^2 - \frac{x}{100} + \frac{1}{40000}\right) - \frac{1}{40000} - 1$

$\left(\quad - \frac{1}{\quad}\right)^2$

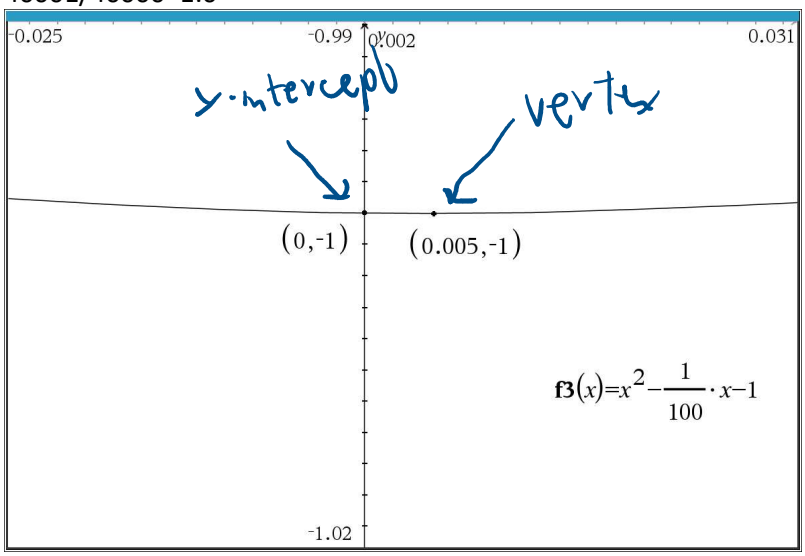
$f(x) = \left(x - \frac{1}{200}\right)^2 - \frac{40000}{40000} - \frac{1}{40000}$

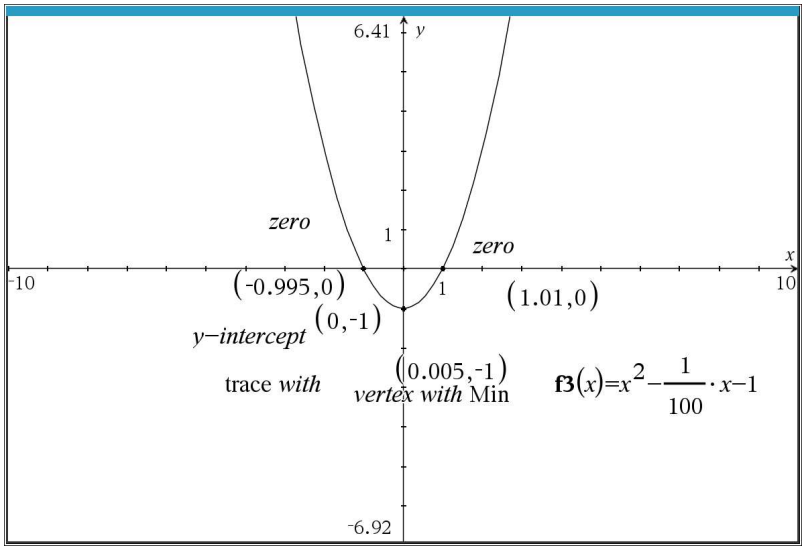
$f(x) = \left(x - \frac{1}{200}\right)^2 - \frac{40001}{40000}$

vertex $= \left(\frac{1}{200}, -\frac{40001}{40000}\right)$

1/200=0.005

40001/40000=1.0





Domain = $(-\infty, \infty)$

Domain = $(-\infty, \infty)$

Range = $\left[-\frac{40001}{40000}, \infty\right)$

decreasing $(-\infty, 0.005)$

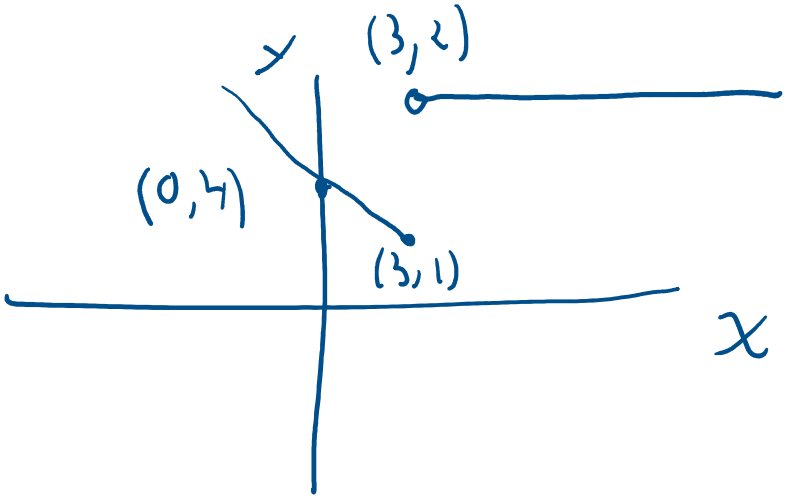
increasing $(0.005, \infty)$

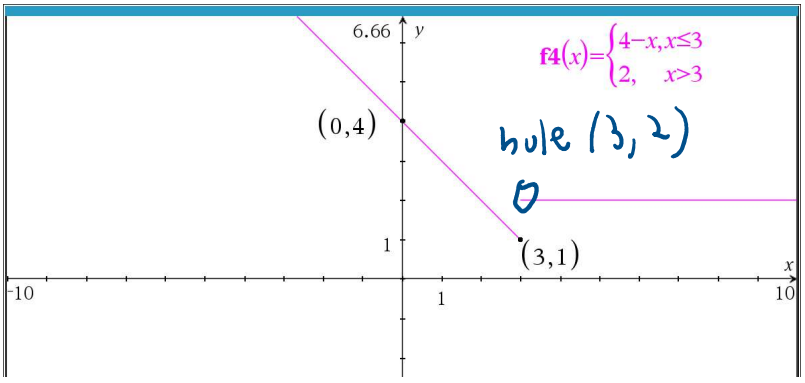
local and global min $= -\frac{40001}{40000}$ at $x = 0.005$

1.6

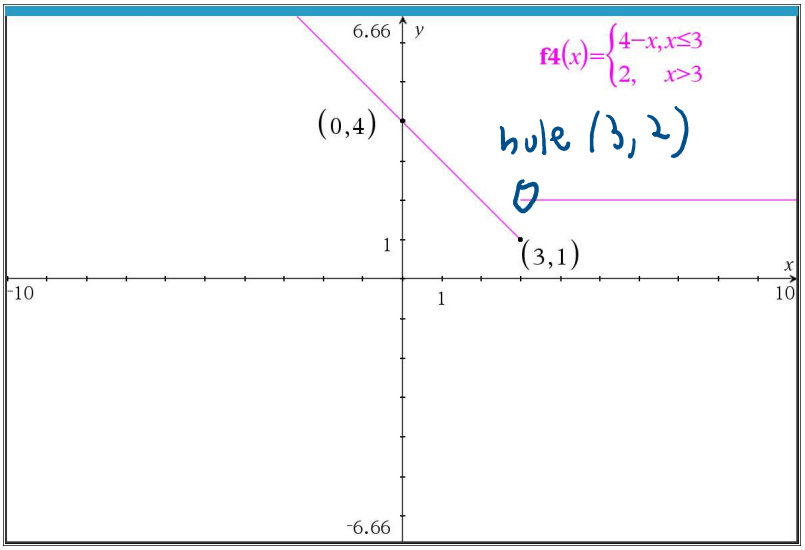
In Exercises 13 - 20, sketch the graph of the given piecewise-defined function.

13. $f(x) = \begin{cases} 4 - x & \text{if} \quad x \leq 3 \\ 2 & \text{if} \quad x > 3 \end{cases}$





$Y_1 = (4 - x) \cdot (x \leq 3) + 2(x > 3)$



$Y_1 = (4-x)\cdot(x \le 3) + 2(x > 3)$