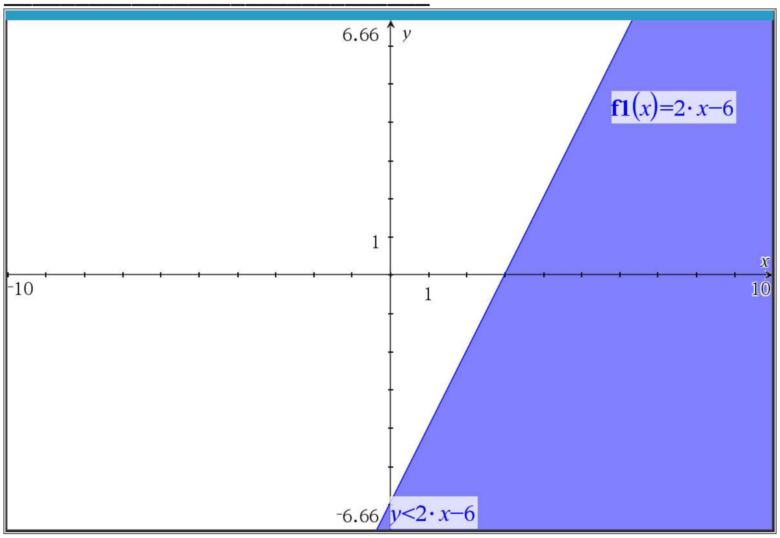
03-06-25 MDE 61

2.4 Inequalities with Absolute Value and Quadratic Functions

2.4.1 Exercises
page 220: 1, 8, 17, 34, 36

Exam 2
Thursday, 03/13/25 (changed from Wednesday)
1.6-1.7,2.1 -2.4



Exam

#2 add

$2x + 5x$

?

$\boxed{6x}$

scratch

$$\begin{array}{r} 2 \\ +\ 5 \\ \hline 7 \end{array}$$

$2+2=8$

? $7x$

Steps for Solving a Quadratic Inequality

1. Rewrite the inequality, if necessary, as a quadratic function $f(x)$ on one side of the inequality and 0 on the other.

2. Find the zeros of f and place them on the number line with the number 0 above them.

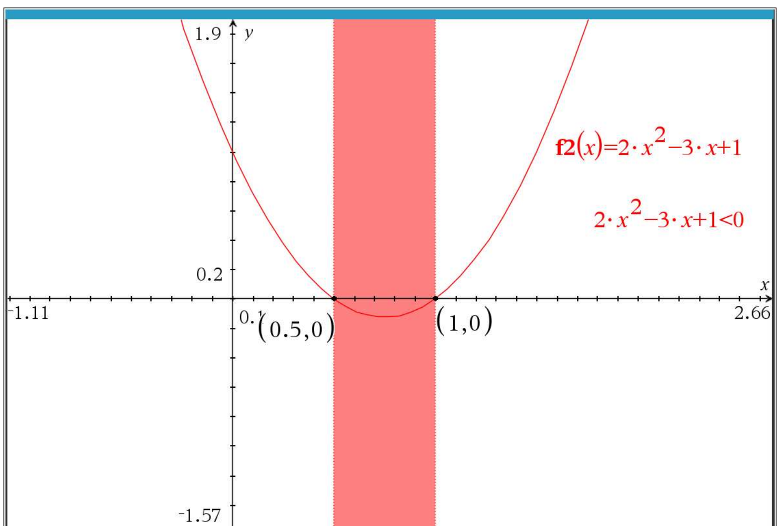
Steps for Solving a Quadratic Inequality

1. Rewrite the inequality, if necessary, as a quadratic function $f(x)$ on one side of the inequality and 0 on the other.
2. Find the zeros of f and place them on the number line with the number 0 above them.
3. Choose a real number, called a **test value**, in each of the intervals determined in step 2.
4. Determine the sign of $f(x)$ for each test value in step 3, and write that sign above the corresponding interval.
5. Choose the intervals which correspond to the correct sign to solve the inequality.

Numeric test value method

Solve $f(x) < 0$

for $f(x) = 2x^2 - 3x + 1$



graphical help

graph solution $(0.5, 1)$

solve $2x^2 - 3x + 1 = 0$

$(2x - 1)(x - 1) = 0$

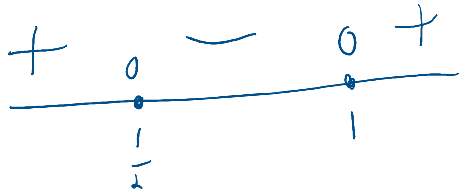
$2x - 1 = 0$ or $x - 1 = 0$

$2x = 1$

$\boxed{x = \frac{1}{2}}$ or $\boxed{x = 1}$

exact solution $\left(\frac{1}{2}, 1\right)$

$\boxed{\text{exact solution } \left(\tfrac{1}{2}, 1\right)}$



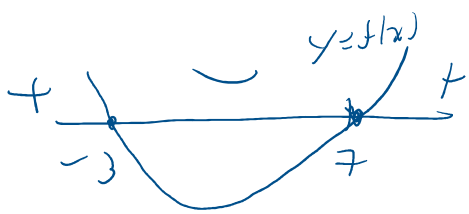
$f(0) = 2(0^2) - 3(0) + 1 = 0 - 0 + 1 = 1 > 0$

$$f\left(\tfrac{3}{4}\right) = 2\left(\tfrac{3}{4}\right)^2 - 3\left(\tfrac{3}{4}\right) + 1$$
$$= 2\left(\tfrac{9}{16}\right) - \tfrac{9}{4} + 1$$
$$= \tfrac{9}{8} - \tfrac{9}{4} + 1$$
$$= 9\left(\tfrac{1}{8} - \tfrac{1}{4}\right) + 1$$
$$= 9\left(\tfrac{1}{8} - \tfrac{2}{8}\right) + 1$$
$$= -\tfrac{9}{8} + 1 < 0$$

$$f(2) = 2(2^2) - 3(2) + 1$$
$$= 8 - 6 + 1$$
$$= 3 > 0$$

---

solve $f(x) = (x-7)(x+3) \leq 0$



coeff of x^2 is positive

$\Rightarrow$ parabola opens up

solution $[-3, 7]$

---

algebraic method

$$(x-7)(x+3) \leq 0$$

$x-7 \le 0$ and $x+3 \ge 0$ or $x-7 \ge 0$ and $x+3 \le 0$

$x \le 7$ and $x \ge -3$ | $x \ge 7$ and $x \le -3$

no solution

or

$\boxed{[-3, 7]}$

2.4:36

36. The height h in feet of a model rocket above the ground t seconds after lift-off is given by $h(t) = -5t^2 + 100t$, for $0 \le t \le 20$. When is the rocket at least 250 feet off the ground? Round your answer to two decimal places.

$h(t) \ge 250$

$-5t^2 + 100t \ge 250$

$-5t^2 + 100t - 250 \ge 0$

$\frac{-5t^2}{-5} + \frac{100t}{-5} - \frac{250}{-5} \ge \frac{0}{-5}$

$f(t) = t^2 - 20t + 50 \le 0$ $\qquad \boxed{0 \le t \le 20}$

$h(t) \ge 250 \iff f(t) \le 0$

solve $t^2 - 20t + 50 = 0$

$t = \frac{20 \pm \sqrt{400 - 4(1)(50)}}{2}$

$t = \frac{20 \pm \sqrt{400 - 200}}{2}$

$t = \frac{20 \pm \sqrt{200}}{}$

$$\frac{\quad}{2}$$

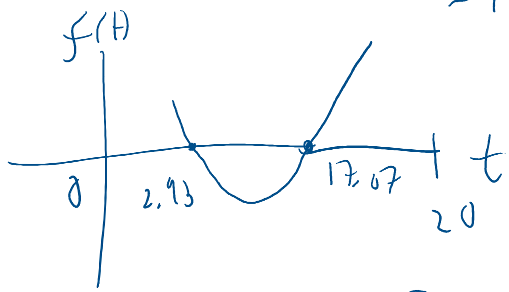
$$t = \frac{20 \pm \sqrt{100 \cdot 2}}{2}$$

$$t = \frac{20 \pm \sqrt{100}\sqrt{2}}{2}$$

$$t = \frac{20 \pm 10\sqrt{2}}{2}$$

$$\boxed{t = 10 \pm 5\sqrt{2}}$$

10-5*sqrt(2)=2.928932188134524 ≈ 2.93
10+5*sqrt(2)=17.07106781186548 ≈ 17.07

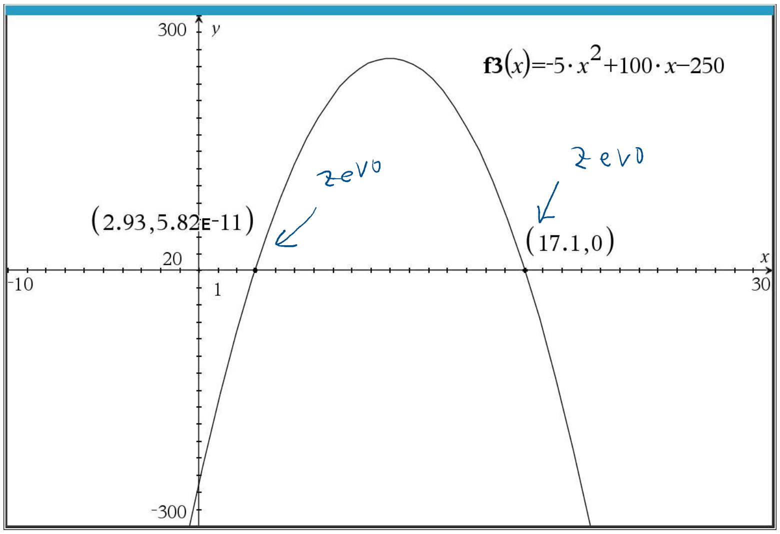


$$2.93 \le t \le 17.07$$

The rocket is at least 250 feet above the ground between 2.93 seconds and 17.07 seconds after lift-off.

After class notes

2.4: 36 use calculator only to produced numerical estimate.



The rocket is at least 250 feet above the ground between 2.93 seconds and 17.1 seconds after lift-off.

Convert $f(x) = 5x^2 + 10x - 9$

to vertex form (standard) $a(x-h)^2 + k = f(x)$

vertex $= (h, k)$

complete the square

① group x terms together

$f(x) = (5x^2 + 10x) - 9$

② factor out the coefficient of x^2

$f(x) = 5(x^2 + 2x) - 9$

③ $\left(\frac{1}{2}\right)$ coefficient of x

square it

add and subtract it inside parentheses

$\left(\frac{1}{2}\right)(2) = 1$

$1^2 = 1$

$f(x) = 5(x^2 + 2x + 1 - 1) - 9$

④ Take negative constant

(4) Take negative constant out of the parentheses

$f(x) = 5(x^2 + 2x + 1) - 5 - 9$

$a(b + c)$
$= ab + ac$

(5) factor the perfect square

$\boxed{f(x) = 5(x + 1)^2 - 14}$

$f(x) = 5(x - (-1))^2 - 14$

vertex $(-1, -14)$