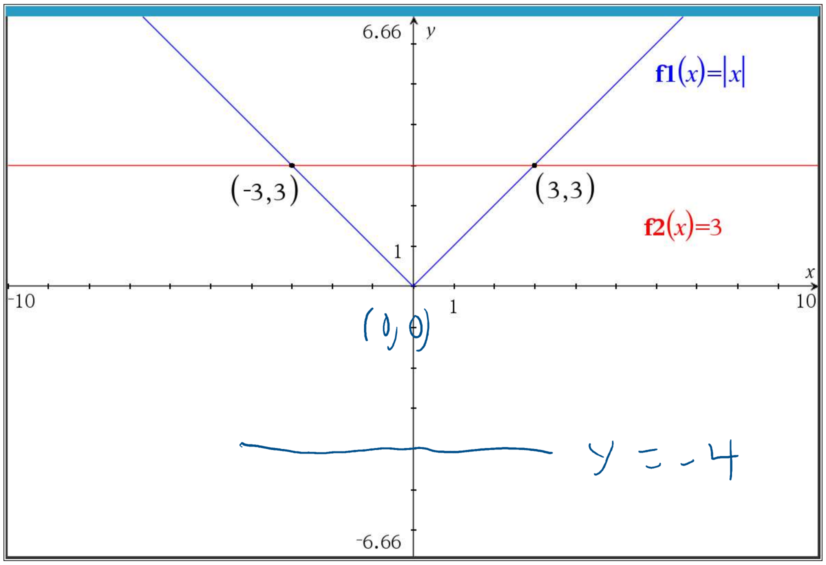
02-27-25 MDE 61

2.2 Absolute Value Functions

2.2.1 Exercises

page 183: 1, 2, 15, 17, 22, 29



$|x| = 3$

$\boxed{x = \pm 3}$ 2 distinct real solutions

$|x| = 0$

$x = \pm 0$

$\boxed{x = 0}$ 1 real solution

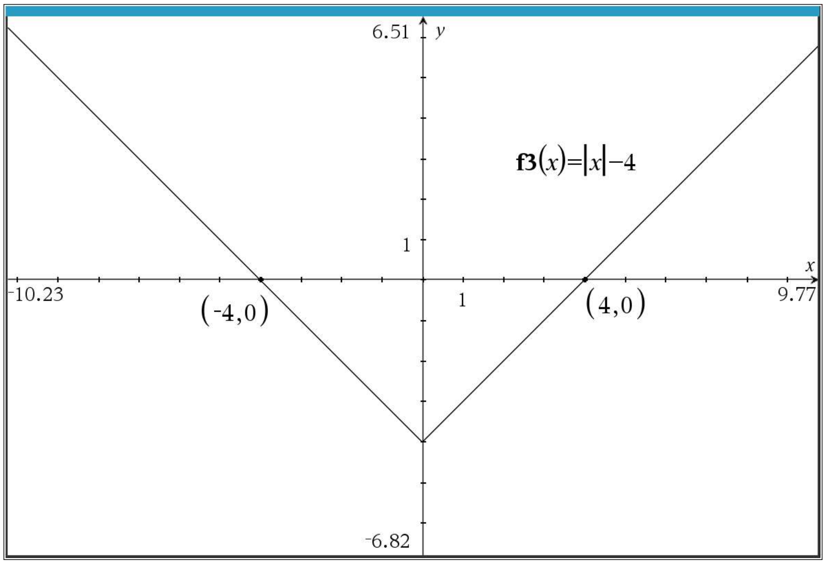
$|x| = -4$

No solution because $|x| \geq 0$

solve $|x| - 4 = 0$

$|x| = 4$

$x = \pm 4$



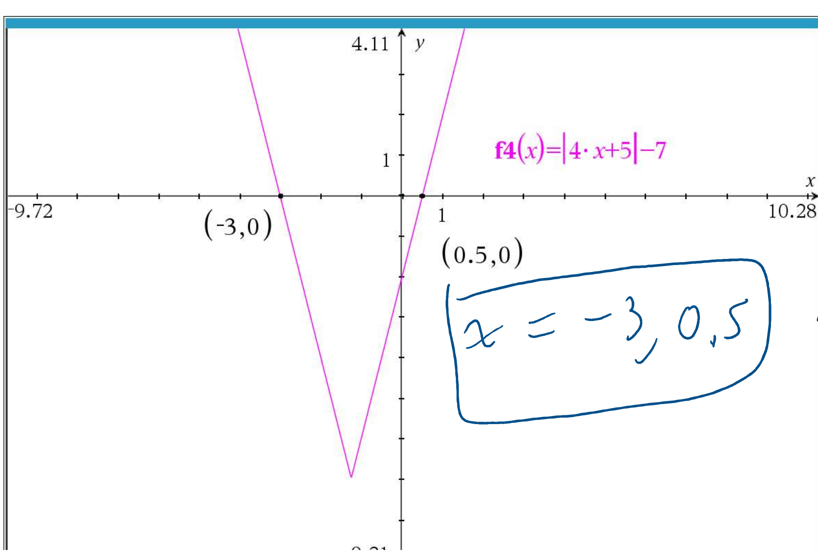
solve $|4x+5| = 7$

$4x+5 = \pm 7$

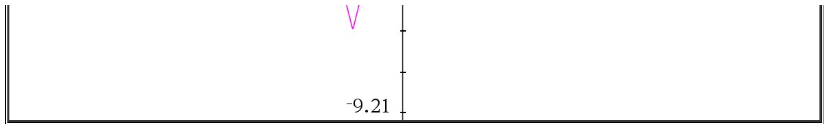
$4x+5 = -7$ or $4x+5 = 7$

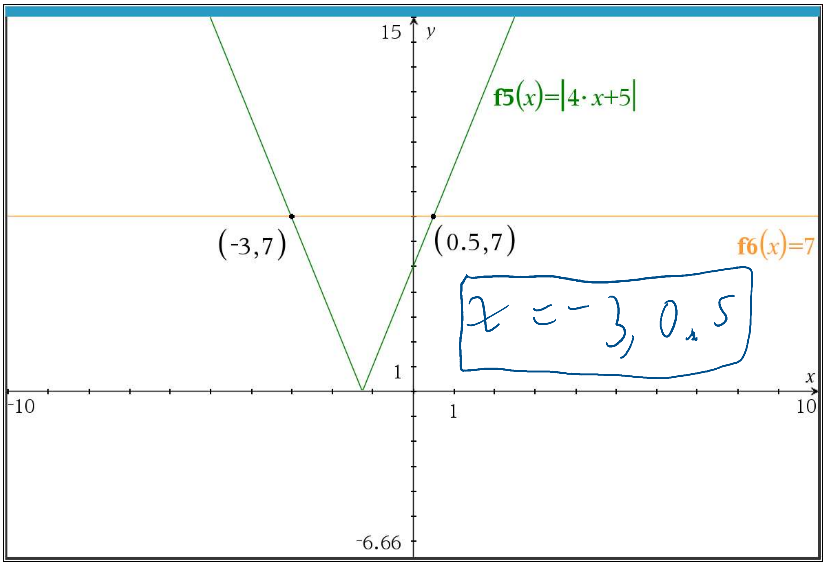
$4x = -12$ $\qquad$ $4x = 2$

$\boxed{x = -3}$ $\qquad$ $\boxed{x = \frac{1}{2}}$ $\qquad$ $\frac{1}{2} = 0,5$



$\boxed{x = -3, 0,5}$





2.2

In Exercises 22 - 33, graph the function. Find the zeros of each function and the x- and y-intercepts of each graph, if any exist. From the graph, determine the domain and range of each function, list the intervals on which the function is increasing, decreasing or constant, and find the relative and absolute extrema, if they exist.

27. $f(x) = \dfrac{1}{3}|2x - 1|$

solve $\frac{1}{3}|2x-1| = 0$

$|2x-1| = 0$

$2x - 1 = \pm 0$

$2x - 1 = 0$

$2x = 1$

$\boxed{x = \frac{1}{2}}$ x-intercept

y-intercept $f(0) = \frac{1}{3}|2(0) - 1|$

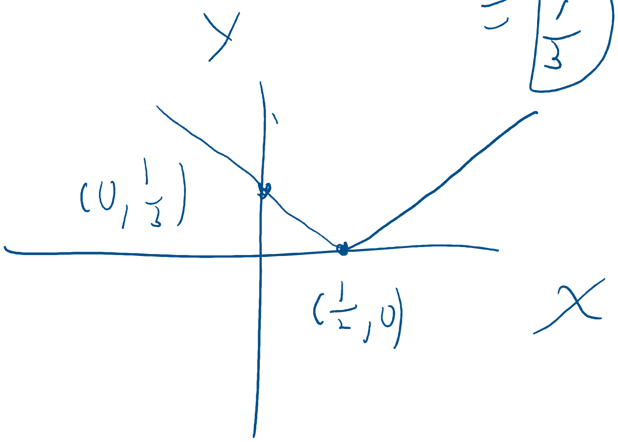
y-intercept $f(0) = \frac{1}{3}|2(0) - 1|$

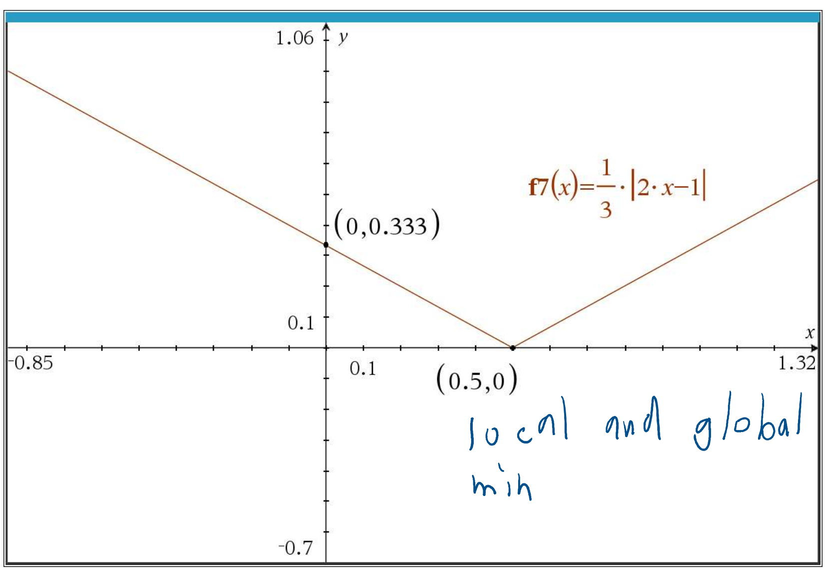
$$= \frac{1}{3}|0 - 1|$$

$$= \frac{1}{3}|-1|$$

$$= \frac{1}{3}(1)$$

$$= \frac{1}{3} \text{ y-intercept}$$





domain $= \mathbb{R} = (-\infty, \infty)$

range $= [0, \infty)$

Increasing $(0.5, \infty)$

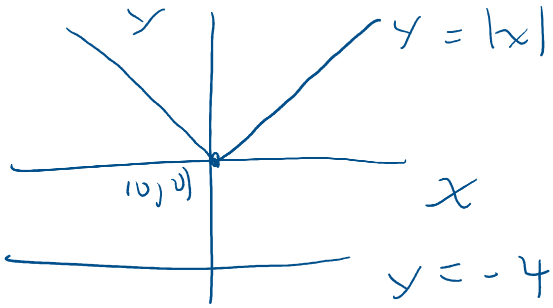
decreasing $(-\infty, 0.5)$

23. $f(x) = |x| + 4$

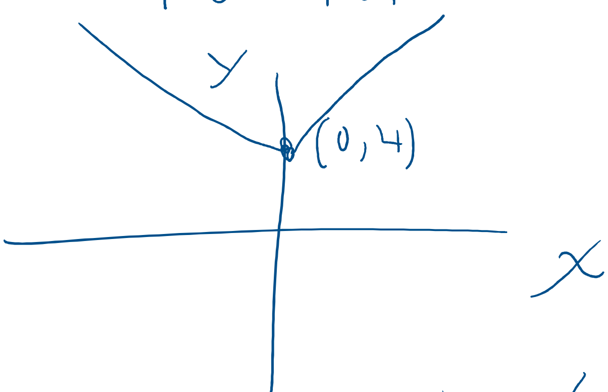
Find zeros, x-intercept, y-intercept if they exist

$|x| + 4 = 0$

$|x| = -4$ No solution, no x-intercept



$f(0) = |0| + 4 = 4$ y-intercept



domain $= \mathbb{R} = (-\infty, \infty)$

local and global min $(0, 4)$

increasing $(0, \infty)$

decreasing $(-\infty, 0)$

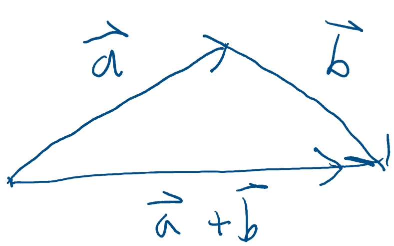
range $[4, \infty)$

36. Prove **The Triangle Inequality:** For all real numbers a and b, $|a + b| \leq |a| + |b|$.

extra

$\vec{a}$ = vector

$|\vec{a}|$ = length of vector



$\vec{a} + \vec{b}$

$$|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$$

Consider example

$a = b = 0$

$$|0 + 0| \overset{?}{\le} |0| + |0|$$

$$|0| \overset{?}{\le} 2|0|$$

$$0 \overset{?}{\le} 2(0)$$

$$0 \le 0 \ \checkmark$$

$a = b = 1$

$$|1 + 1| \overset{?}{\le} |1| + |1|$$

$$|2| \overset{?}{\le} 1 + 1$$

$$2 \le 2 \ \checkmark$$

$a = 2, \quad b = 3$

$$|2 + 3| \overset{?}{\le} |2| + |3|$$

$$|5| \overset{?}{\le} 2 + 3$$

$$5 \overset{?}{\le} 2 + 3$$

$$5 \le 5 \ \checkmark$$

$a = 1, \quad b = -2$

$a = 1, \ b = -2$

$$|1 - 2| \overset{?}{\leq} |1| + |-2|$$

$$|-1| \overset{?}{\leq} 1 + 2$$

$$1 \overset{?}{\leq} 3 \quad \text{True}$$

in fact $1 < 3$