02-20-25 MDE 61

1.7 Transformations

1.7.1 Exercises

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Your Name MDE 61 quiz 2 calculator required

1. Find and simplify the difference quotient for $f(x) = -2x + 3$.

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{[-2(x+h)+3] - [-2x+3]}{h}$$

distributive property of multiplication over addition and subtraction

$$= \frac{\cancel{-2x} - 2h \cancel{+3} + \cancel{2x} \cancel{-3}}{h}$$

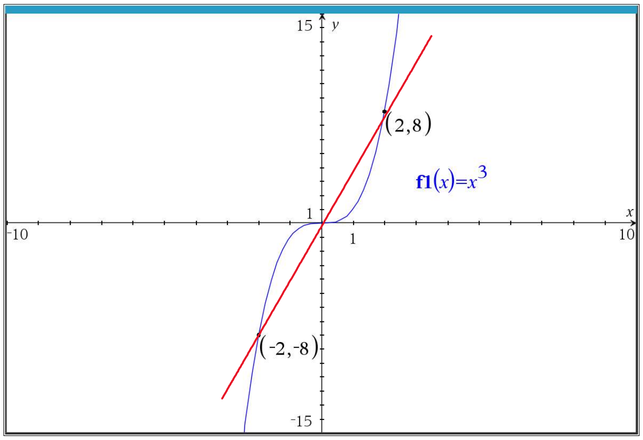
$$= \frac{-2\cancel{h}}{\cancel{h}} = \boxed{-2}$$

2. Is $f(x) = x^3$ even, odd, or neither? Show the calculation that justifies your answer. Show a graph from your calculator, plotting points that illustrate your answer.

$$f(-x) = (-x)^3 = (-1)^3 x^3 = -x^3 = -f(x)$$

$\Rightarrow$ f is odd

To be even, we need $f(-x) = f(x)$, but this is $f(x)$



The fact that (2,8) and (-2,-8) are both points on the graph supports the idea that the function is odd.

I used Trace with x =2 and x = -2 to find the points.

3. Is every relation a function? Why or why not?

   No. Let $R = \{(1,3), (1,4)\}$

R is a relation, that is, R is a set of points in the plane, or R is set of ordered pairs of real numbers.

However, R does not represent y as a function of x, because the input 1 has two outputs, namely 3 and 4.

1.7

1.7.1 Exercises

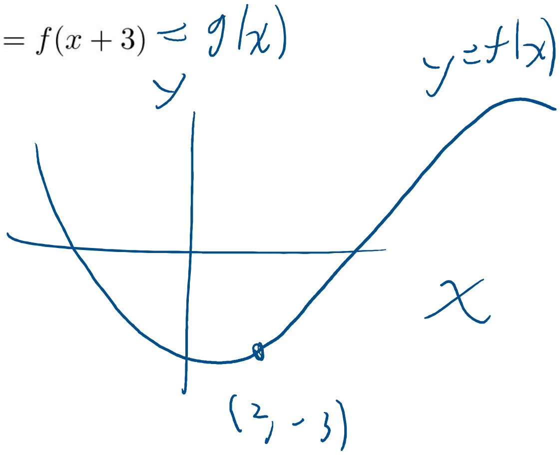
Suppose $(2, -3)$ is on the graph of $y = f(x)$. In Exercises 1 - 18, use Theorem 1.7 to find a point on the graph of the given transformed function.

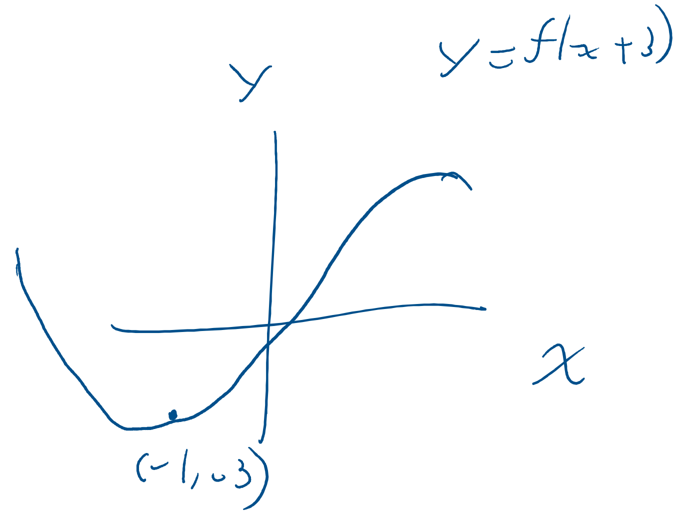
Theorem 1.7. Transformations. Suppose f is a function. If $A \neq 0$ and $B \neq 0$, then to graph

$$g(x) = Af(Bx + H) + K$$

1. Subtract H from each of the x-coordinates of the points on the graph of f. This results in a horizontal shift to the left if $H > 0$ or right if $H < 0$.
2. Divide the x-coordinates of the points on the graph obtained in Step 1 by B. This results in a horizontal scaling, but may also include a reflection about the y-axis if $B < 0$.
3. Multiply the y-coordinates of the points on the graph obtained in Step 2 by A. This results in a vertical scaling, but may also include a reflection about the x-axis if $A < 0$.
4. Add K to each of the y-coordinates of the points on the graph obtained in Step 3. This results in a vertical shift up if $K > 0$ or down if $K < 0$.

2. $y = f(x+3) = g(x)$





$$g(x) = Af(Bx + H) + K$$

$A = 1$ $\quad B = 1$ $\quad H = 3$ $\quad K = 0$

① $(2, -3)$

$(2-3, -3)$

$(-1, -3)$

② $\left(-\frac{1}{1}, -3\right)$

$(-1, -3)$

③ $(-1, (-3)(1))$

$(-1, -3)$

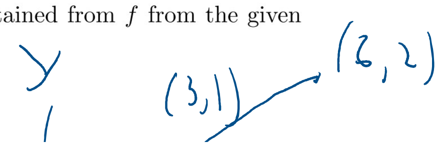
④ $(-1, -3+0)$

$(-1, -3)$

1.7

Let $f(x) = \sqrt{x}$. Find a formula for a function g whose graph is obtained from f from the given sequence of transformations.

54. (1) shift right 2 units; (2) shift down 3 units



sequence of transformations.

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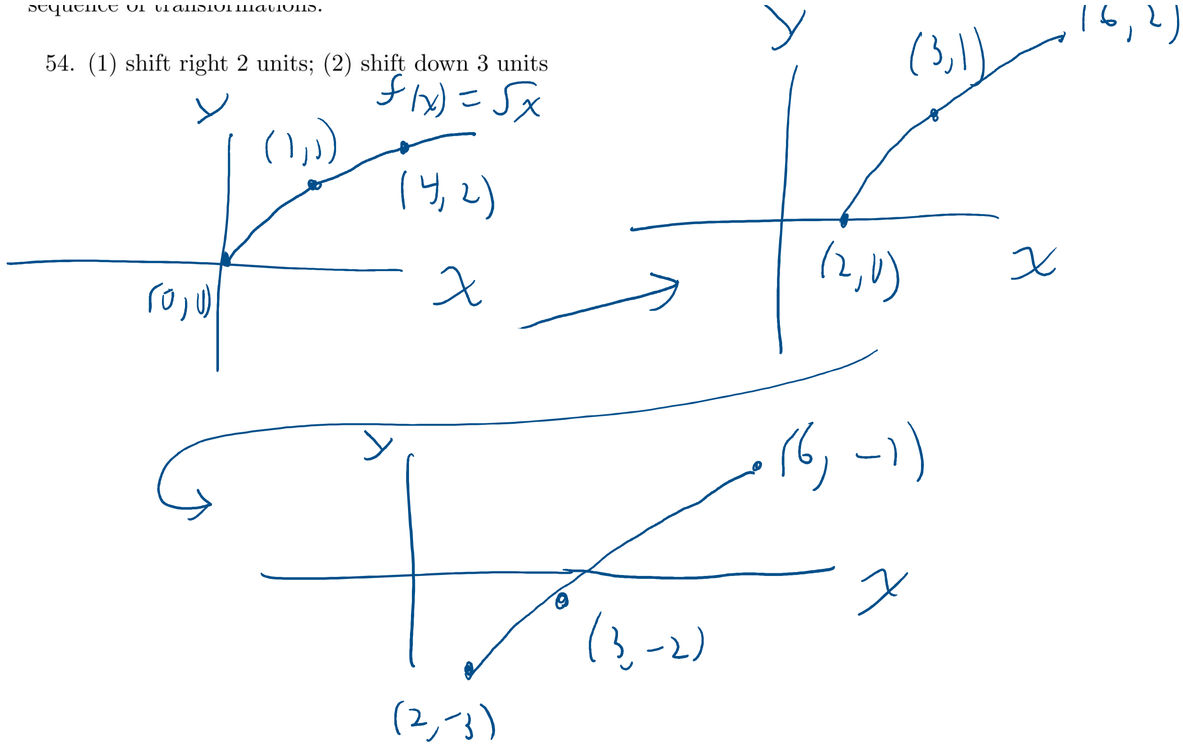
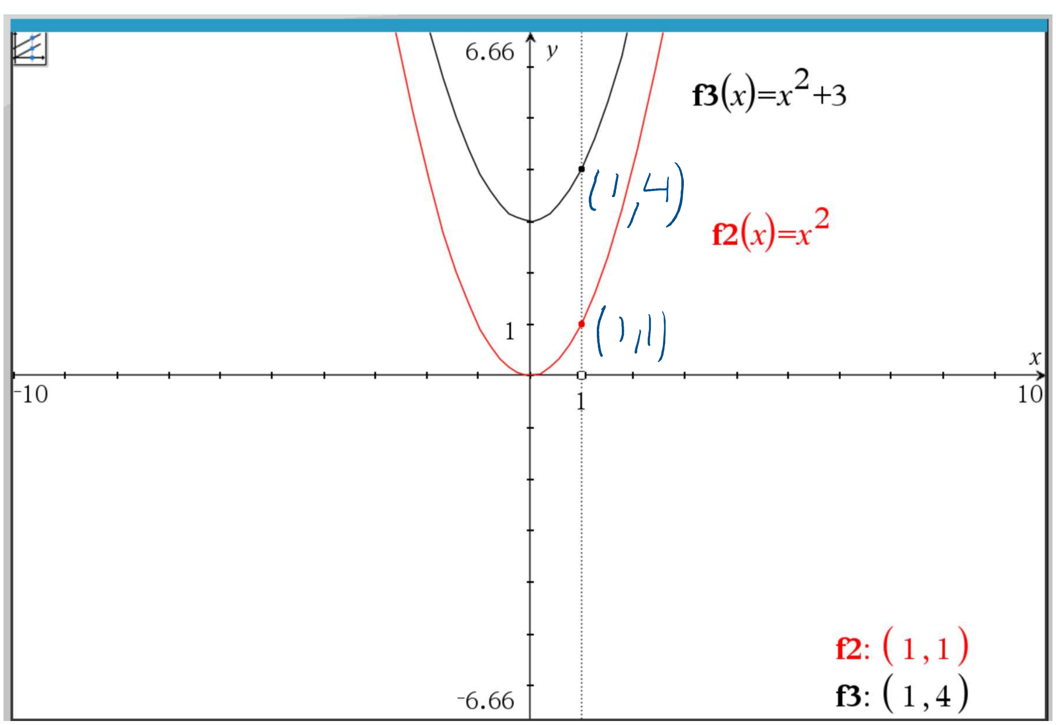
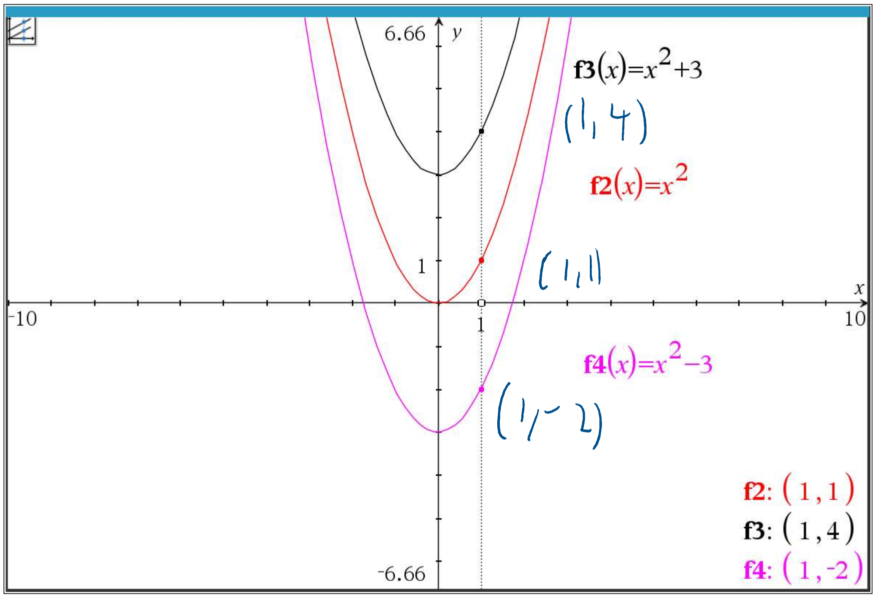


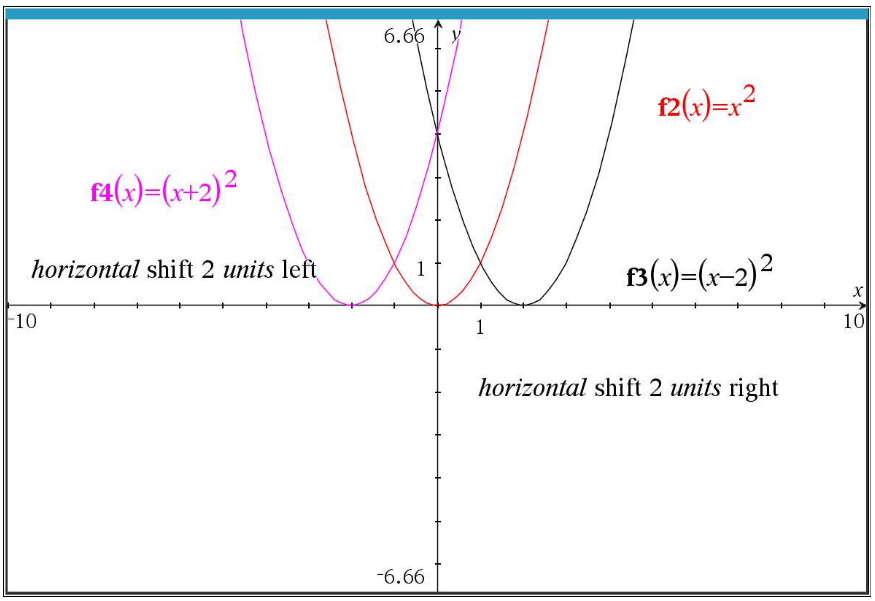
$$f(x) = \sqrt{x}$$

$$f_1(x) = f(x-2) = \sqrt{x-2}$$

$$g(x) = f_1(x) - 3 = \boxed{\sqrt{x-2} - 3}$$







Memorize

Theorem 1.4. Reflections. Suppose f is a function.

- To graph $y = -f(x)$, reflect the graph of $y = f(x)$ across the x-axis by multiplying the y-coordinates of the points on the graph of f by -1.
- To graph $y = f(-x)$, reflect the graph of $y = f(x)$ across the y-axis by multiplying the x-coordinates of the points on the graph of f by -1.

Memorize

Theorem 1.5. Vertical Scalings. Suppose f is a function and $a > 0$. To graph $y = af(x)$, multiply all of the y-coordinates of the points on the graph of f by a. We say the graph of f has been vertically scaled by a factor of a.

- If $a > 1$, we say the graph of f has undergone a vertical stretching (expansion, dilation) by a factor of a.
- If $0 < a < 1$, we say the graph of f has undergone a vertical shrinking (compression, contraction) by a factor of $\frac{1}{a}$.