

1.6 Graphs of Functions

1.6.2 Exercises

page 107: 1, 7, 9, 14, 21, 24, 32, 75

Difference quotient

Find and simplify the difference quotient $\frac{\Delta f}{\Delta x}$ for $f(x) = 8x - 15$.

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}, \quad h \neq 0 \quad \text{memorize}$$

$$= \frac{[8(x+h) - 15] - [8x - 15]}{h}$$

Distributive property
of multiplication over
addition and subtraction

$$= \frac{\cancel{8x} + 8h - \cancel{15} - \cancel{8x} + \cancel{15}}{h}$$

$$= \frac{8h}{h}$$

$$\boxed{\frac{\Delta f}{\Delta x} = 8}$$

Find and simplify $\frac{\Delta f}{\Delta x}$ for $f(x) = 2x^2 - 4$.

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}, \quad h \neq 0$$

$$= \frac{[2(x+h)^2 - 4] - [2x^2 - 4]}{h}$$

$$(x+h)^2 = (x+h)(x+h) = x^2 + 2hx + h^2$$

$$\begin{array}{r} x+h \\ \times \quad x+h \\ \hline hx + h^2 \\ xh + x^2 \\ \hline \end{array}$$

$hx = xh$
commutative property
of multiplication of
real numbers

$$\frac{(x+h) + x^2}{x^2 + 2hx + h^2}$$

of multiplication of
real number

$$\begin{aligned} \frac{\Delta f}{\Delta x} &= \frac{2(x^2 + 2xh + h^2) - 4 - 2x^2 + 4}{h} \\ &= \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{4} - \cancel{2x^2} + \cancel{4}}{h} \\ &= \frac{4xh + 2h^2}{h} \\ &= \frac{\cancel{h}(4x + 2h)}{\cancel{h}} \end{aligned}$$

$$\boxed{\frac{\Delta f}{\Delta x} = 4x + 2h}$$

Find and simplify $\frac{\Delta f}{\Delta x}$ for $f(x) = \sqrt{x}$.

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

= 1 in a complicated
but useful way

$$= \left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \right) \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$$

in a complicated way

$$\begin{aligned}
 & (a+b)(a-b) \\
 & a^2 + ba - ba - b^2 \\
 & = a^2 - b^2
 \end{aligned}$$

$$\begin{aligned}
 & \frac{h}{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})} \\
 & = \frac{h(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} \\
 & = \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} \\
 & = \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} \\
 & = \frac{h}{h(\sqrt{x+h} + \sqrt{x})}
 \end{aligned}$$

$$\frac{\Delta f}{\Delta x} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$a + b \xrightarrow{\text{conjugate}} a - b$$

Factor 60 into prime factor

Def) n is a prime number if its only factors are n and 1

$$2 = 2 \cdot 1$$

$$3 = 3 \cdot 1$$

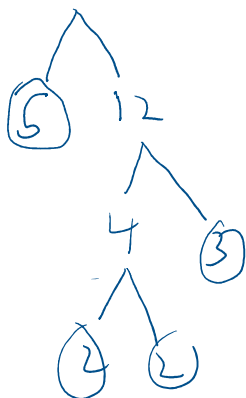
$$5 = 5 \cdot 1$$

$$4 = 4 \cdot 1 = 2 \cdot 2 \text{ (not prime)}$$

composite

$$6 = 6 \cdot 1 = 2 \cdot 3 \text{ (composite)}$$

Factor 60 into prime factors



$$60 = 2^2 \cdot 3 \cdot 5$$

Factor $2x^2 + 4$

$$= 2(x^2 + 2)$$

Factor $3x^2y + 9x$

$$= 3x(xy + 3)$$

Factor $x^2 + 2x + 1$

$$= (x + 1)(x + 1)$$

$$= (x+1)(x+1)$$

$$= (x+1)^2$$

Factor $x^2 + x - 12$

$$= (x+4)(x-3)$$

$$12 = 2 \cdot 6$$

$$\begin{array}{r} 12 \cdot 1 \\ \hline 4 \cdot 3 \end{array}$$

Factor $3x^2 + 27x - 18$

$$3(x^2 + 9x - 6)$$

$$(\cancel{x+7})(\cancel{x-6})$$

$$6 = 3 \cdot 2$$

$$= 6 \cdot 1$$

solve $3x^2 + 27x - 18 = 0$

$$\Leftrightarrow \text{solve } x^2 + 9x - 6 = 0$$

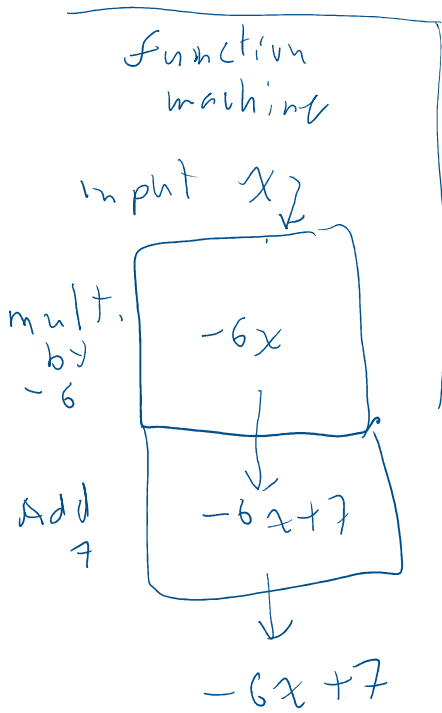
After class notes

Find and simplify $\frac{\Delta f}{\Delta x}$ for $f(x) = -6x + 7$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

function

... .. $f(x+7)$



$$= \frac{[-6(x+h) + 7] - [-6x + 7]}{h}$$

$$= \frac{-6x - 6h + 7 + 6x - 7}{h}$$

$$= \frac{-6h}{h} = -6 \left(\frac{h}{h} \right) = -6(1) = \boxed{-6}$$

$$a(b+c) \stackrel{?}{=} ab + c$$

Let $a=2$
 $b=3$
 $c=1$

$$2(3+1) \stackrel{?}{=} (2)(3) + 1$$

$$(2)(4) \stackrel{?}{=} 6 + 1$$

$$8 \neq 7$$

for any real numbers
 a, b, c

This counterexample shows that the above formula is not true, in general.

Let $a=b=c=0$

$$0(0+0) \stackrel{?}{=} (0)(0) + 0$$

$$0(0) \stackrel{?}{=} 0 + 0$$

$$0 = 0 \quad \checkmark$$

$$\underline{a(b+c) = \underline{ab} + \underline{ac}}$$

Factor $25a^2 - 5a + 15$

$5(5a^2 - a + 3)$ answer

not the same

$3 = 3 \cdot 1$

same? $5(5a - 3)(a - 1)?$ $5(5a-1)(a-3)$

same?

$$5(5a - 3)(a - 1) ?$$

$$5(5a^2 - 3a - 5a + 3)$$

$$5(5a^2 - 8a + 3)$$

\therefore we cannot factor further

~~$$5(sa - 1)(a - 3)$$

$$5(5a^2 - 16a + 3)$$~~

Gemini

◆ Certainly! Here's an example of factoring a trinomial:

Example: Factor the trinomial $x^2 + 5x + 6$.

Steps:

1. **Identify the coefficients:**

- The coefficient of the x^2 term is 1.
- The coefficient of the x term is 5.
- The constant term is 6.

2. **Find two numbers that multiply to the constant term (6) and add up to the coefficient of the x term (5).**

- We need two numbers that multiply to 6 and add to 5.
- The pairs of factors of 6 are: (1, 6) and (2, 3).
- The pair (2, 3) works because $2 * 3 = 6$ and $2 + 3 = 5$.

3. **Write the factored form:**

- Since we found the numbers 2 and 3, the factored form is: $(x + 2)(x + 3)$.

4. **Check (optional):**

- To check, we can expand the factored form:
 - $(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$.
- This matches the original trinomial, so our factoring is correct.

Therefore, the factored form of $x^2 + 5x + 6$ is $(x + 2)(x + 3)$.

Please factor $25a^2 - 5a + 15$.

★ Let's factor the expression $25a^2 - 5a + 15$.

1. Find the Greatest Common Factor (GCF):

- Look for a common factor among the coefficients 25, -5, and 15.
- The greatest common factor of these numbers is 5.

2. Factor out the GCF:

- Divide each term by 5:
 - $25a^2 / 5 = 5a^2$
 - $-5a / 5 = -a$
 - $15 / 5 = 3$
- Write the factored expression: $5(5a^2 - a + 3)$

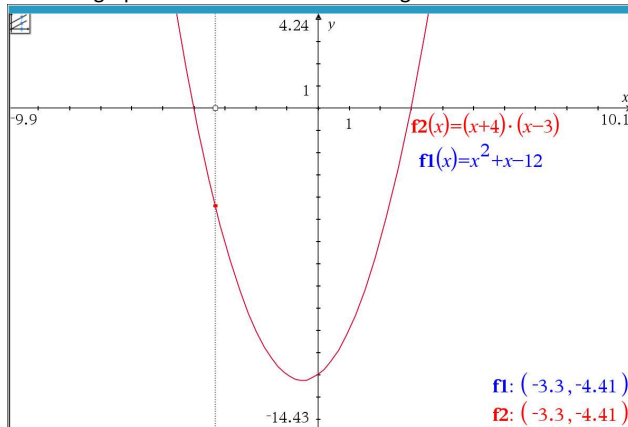
3. Check if the trinomial inside the parentheses can be factored further:

- Now, we need to see if $5a^2 - a + 3$ can be factored.
- To do this we would need to find two numbers that multiply to $(5 \cdot 3) = 15$ and add to -1.
- The factor pairs of 15 are (1,15), (3,5). There is no combination of these factors that will add up to -1.
- Therefore, the trinomial $5a^2 - a + 3$ cannot be factored further using integer coefficients.

Final Answer:

- The factored form of $25a^2 - 5a + 15$ is $5(5a^2 - a + 3)$.

Here is a graphical check that our factoring is correct.



The two graphs exactly overlap.