04-13-16 MTH 263

5.3 The Fundamental Theorem of Calculus
page 155: 1, 4, 7, 11, 12, 13

5.4 Integration by Substitution
page 160: 1, 3, 5, 15, 23, 27

5.4
Rule of thumb for u-substitution: look for messy expressions, expressions inside parentheses, expression under radical signs, expressions in denominators
Then, look to see if du is in your original integrand, possibly multiplied by a constant

$$I = \int e^{-3x}\,dx$$

Let $u = e^{-3x}$

$$du = -3e^{-3x}\,dx$$

$$e^{-3x}\,dx = \frac{du}{-3}$$

$$I = -\frac{1}{3}\int du = -\frac{1}{3}(u) + C$$

$$I = -\frac{1}{3}\left(e^{-3x}\right) + C$$

To use substitution with definite integrals, follow the same procedure as with indefinite integrals but add one extra step: replace the limits of integration $x = a$ and $x = b$ in the original integral $\int_a^b f(x)\,dx$ by $u = g(a)$ and $u = g(b)$, respectively, in the new integral involving u, where $u = g(x)$ is your substitution.

Example 5.26

Evaluate $\displaystyle\int_1^2 (2x+1)^3\,dx$. $= I$

Solution: Let $u = g(x) = 2x+1$, which means that $dx = \frac{1}{2}\,du$. The upper limit of integration $x = 2$ becomes $u = g(2) = 2(2)+1 = 5$ in the new u-based integral, while the lower limit of integration $x = 1$ becomes $u = g(1) = 2(1)+1 = 3$. Thus:

$$\int_1^2 (2x+1)^3\,dx \;=\; \frac{1}{2}\int_3^5 u^3\,du \;=\; \frac{1}{8}u^4\,\Big|_3^5 \;=\; \frac{1}{8}(5^4 - 3^4) \;=\; 68$$

Note that you could have put everything back in terms of x at the end, but there was no need to since you would get the same numerical answer.

$u = 2x+1$

$\Rightarrow du = 2\,dx \Rightarrow dx = \dfrac{du}{2}$

$I = \displaystyle\int_{x=1}^{2} u^3\,du$

$= \left.\dfrac{u^4}{4}\right]_{x=1}^{2}$

$= \left.\dfrac{(2x+1)^4}{4}\right]_1^2$

$= \dfrac{5^4}{4} - \dfrac{3^4}{4}$

$= \boxed{68}$

Supplied

For any constant a,

$$\int_0^a f(x)\,dx \;=\; \int_0^a f(a-x)\,dx\,. \; = I \qquad (5.6)$$

Let $u = a - x$

Let $u = a - x$

$\Rightarrow du = -dx \Rightarrow dx = -du$

$x = 0 \Rightarrow u = a$

$x = a \Rightarrow u = 0$

$I = -\int_a^0 f(u)\, du$

$= \int_0^a f(u)\, du$ rename the dummy variable u

$= \int_0^a f(x)\, dx$

This is simple to prove, using the substitution $u = a - x$, so $x = a - u$ and $dx = -du$, while $x = 0$ becomes $u = a$ and $x = a$ becomes $u = 0$ in the limits of integration:

$$\int_0^a f(x)\, dx = -\int_a^0 f(a-u)\, du = \int_0^a f(a-u)\, du = \int_0^a f(a-x)\, dx \quad \checkmark$$

Your Name MTH 263 quiz 6 write each problem. Need calculator.

1. Evaluate $I = \int \frac{2x}{x^2+5}\, dx$.

Let $u = x^2 + 5$

$\Rightarrow du = 2x\, dx$

$\Rightarrow I = \int \frac{du}{u} = \ln|u| + C = \boxed{\ln|x^2+5| + C}$

2. Evaluate $I = \int_{-2}^{2} x^3\, dx$ in two ways, without a calculator.

$(-x)^3 = -x^3 \Rightarrow x^3$ odd function

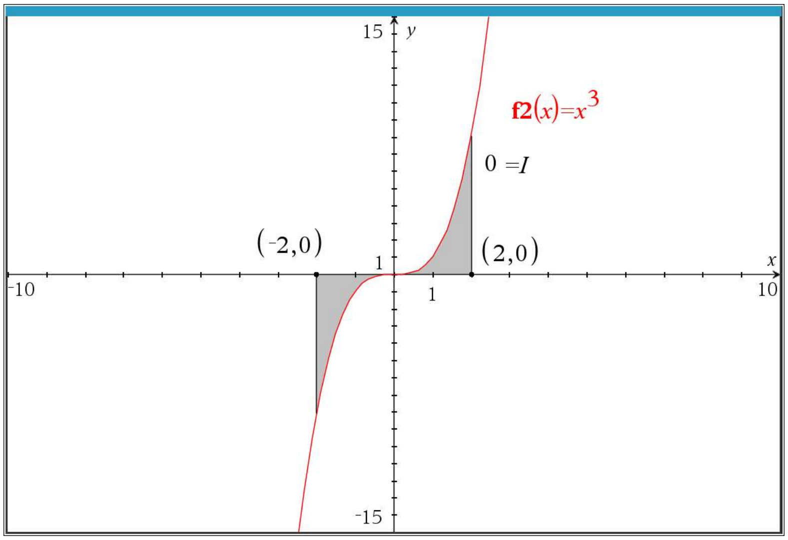
limits of integration of form $-a, a$

$\therefore \boxed{I = 0}$

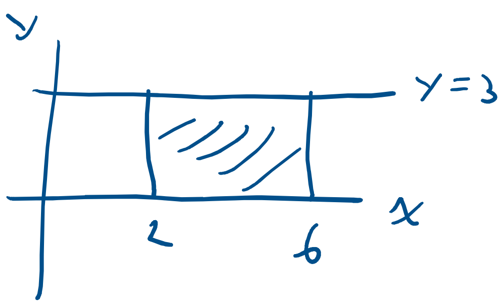
$I = \left.\frac{x^4}{4}\right]_{-2}^{2} = \frac{2^4}{4} - \frac{(-2)^4}{4} = \frac{2^4}{4} - \frac{2^4}{4} = \boxed{0}$

$I = \frac{x}{4}\Big|_{-2} = \frac{\ }{4} - \frac{(\ )}{4} = \frac{\ }{4} - \frac{\ }{4} = \boxed{\ }$

3. Evaluate $I = \int_{-2}^{2} x^3\, dx$ graphically. Sketch a labeled graph.



4. Evaluate $I = \int_{2}^{6} 3\ dx$ geometrically.



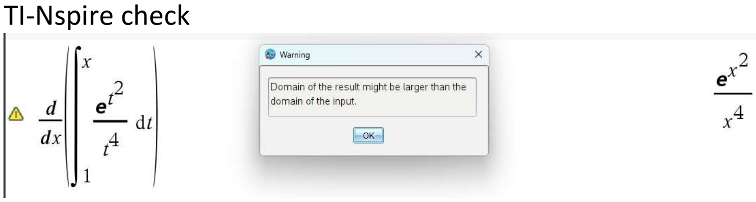
$I =$ area of rectangle
$= (6-2)(3)$
$= (4)(3) = \boxed{12}$

5. Let $A(x) = \displaystyle\int_1^x \frac{e^{t^2}}{t^4}\ dt$. Calculate $A'(x)$.

By Fund. Thm of Calc

$A'(x) = \dfrac{e^{x^2}}{x^4}$

TI-Nspire check



TI could not evaluate the integral.

$$\int_1^x \frac{e^{t^2}}{t^4}\,dt \qquad \int_1^x \frac{e^{t^2}}{t^4}\,dt$$

TI could only estimate the definite integral.

$$\int_1^2 \frac{e^{t^2}}{t^4}\,dt \qquad 2.23061$$