03-25-26 MTH 263

3.6 Differentials
page 98: 1, 3, 5, 9

4 Applications of Derivatives
4.1 Optimization
page 108: 1, 3, 7, 13, 17

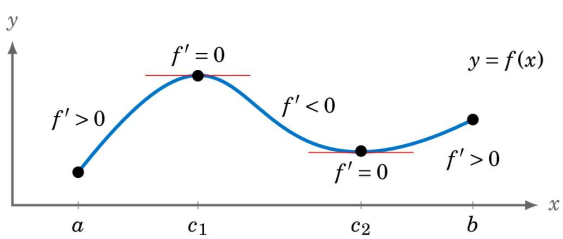
10 textbook sections remaining
12 class meetings before final exam
1 section per class meeting

4.1
Memorize

A function f has a **global maximum** at $x = c$ if $f(c) \geq f(x)$ for all x in the domain of f. Similarly, f has a **global minimum** at $x = c$ if $f(c) \leq f(x)$ for all x in the domain of f. Say that f has a **local maximum** at $x = c$ if $f(c) \geq f(x)$ for all x "near" c, i.e. for all x such that $|x - c| < \delta$ for some number $\delta > 0$. Likewise, f has a **local minimum** at $x = c$ if $f(c) \leq f(x)$ for all x such that $|x - c| < \delta$ for some number $\delta > 0$.

Global = absolute
Local = relative

Local max = top of small hill
Local min = bottom of small valley



f' goes from $+$ to 0 to $-$
$\Rightarrow f'$ is decreasing
$\Rightarrow f'' < 0$
$\therefore f(c_1)$ is a local max

f' goes from $-$ to 0 to $+$
$\Rightarrow f'$ " increasing
$\Rightarrow f'' > 0$
$f(c_2)$ is a local min

2nd deriv test

If $f'(w)$ does not exist
but $f' > 0$ to the left of w
$f' < 0$ " " right " "
Then $f(w)$ is a local max

1st deriv. test

Memorize

Second Derivative Test: Let $x = c$ be a critical point of f (i.e $f'(c) = 0$). Then:
(a) If $f''(c) > 0$ then f has a local minimum at $x = c$

Second Derivative Test: Let $x = c$ be a critical point of f (i.e $f'(c) = 0$). Then:
(a) If $f''(c) > 0$ then f has a local minimum at $x = c$.

(b) If $f''(c) < 0$ then f has a local maximum at $x = c$.

(c) If $f''(c) = 0$ then the test fails.

Memorize

How to find a global maximum or minimum
Suppose that f is defined on an interval I. There are two cases:

1. **The interval I is closed**: The global maximum of f will occur either at an interior local maximum or at one of the endpoints of I whichever of these points provides the largest value of f will be where the global maximum occurs. Similarly, the global minimum of f will occur either at an interior local minimum or at one of the endpoints of I; whichever of these points provides the smallest value of f will be where the global minimum occurs.

2. **The interval I is not closed and has only one critical point**: If the only critical point is a local maximum then it is a global maximum. If the only critical point is a local minimum then it is a global minimum.

4.1

4. The power output P of a battery is given by $P = VI - RI^2$, where I, V, and R are the current, voltage, and resistance, respectively, of the battery. If V and R are constant, find the current I that maximizes P.

Second Derivative Test: Let $x = c$ be a critical point of f (i.e $f'(c) = 0$). Then:
(a) If $f''(c) > 0$ then f has a local minimum at $x = c$.

(b) If $f''(c) < 0$ then f has a local maximum at $x = c$.

(c) If $f''(c) = 0$ then the test fails.

Assume $V, R > 0$

$P = VI - RI^2$

$\frac{dP}{dI} = V - 2RI = 0$

$2RI = V$

$\boxed{I = \frac{V}{2R}}$ critical point

$\frac{d^2P}{dI^2} = -2R < 0$

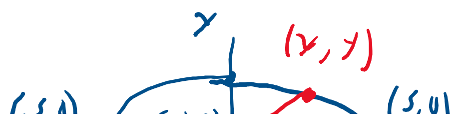
$\therefore$ local max at $I = \frac{V}{2R}$

$0 \leq I$ not a closed interval

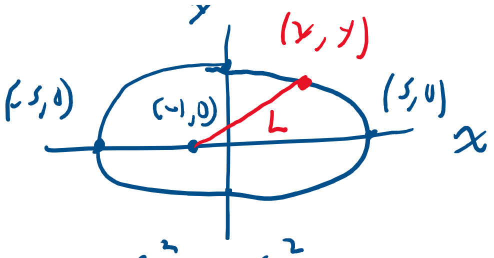
$\therefore$ local max = global max

4.1: 8

8. Find the point(s) on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ closest to $(-1, 0)$.



Let $L = \text{dist}((-1,0),(x,y))$



Let $L = \text{dist}\big((-1,0),(x,y)\big)$

on ellipse

$$\frac{x^2}{25} + \frac{0^2}{16} = 1$$

$$x^2 = 25$$

$$x = \pm 5$$

Find (x,y) to minimize L

Possible Answer form

(A) 5

(B) Virginia

(C) x^2+y^2

$\longrightarrow$ (D) $(8,9)$

$$L = \sqrt{(x-(-1))^2 + (y-0)^2}$$

$$\boxed{L(x,y) = \sqrt{(x+1)^2 + y^2}}$$

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

solve for y

$$\frac{y^2}{16} = 1 - \frac{x^2}{25}$$

$$y^2 = 16 - \left(\frac{16}{25}\right) x^2$$

$$L(x) = \sqrt{(x+1)^2 + 16 - \left(\frac{16}{25}\right) x^2}$$

Let's minimize $L^2(x)$.

This gives the same critical point(s)

$$L^2(x) = x^2 + 2x + 1 + 16 - \left(\frac{16}{25}\right) x^2$$

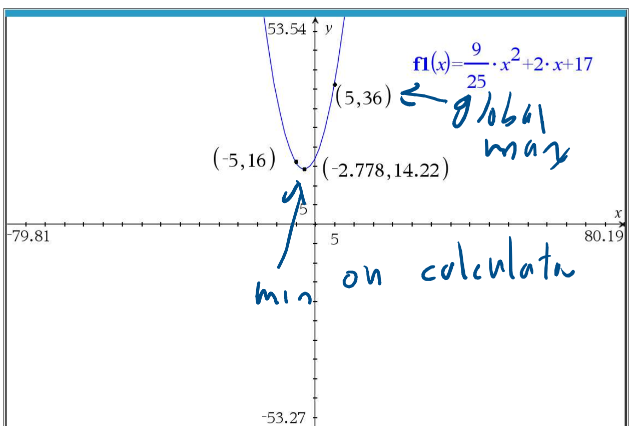
$$L^2(x) = x^2\left(1 - \frac{16}{25}\right) + 2x + 17$$

$$\boxed{L^2(x) = \left(\frac{9}{25}\right) x^2 + 2x + 17}$$

take 1st derivative to find critical point

Find critical point and its value

Then compare with $L^2(-5)$ and $L^2(5)$



$$\boxed{(x, y) \approx (-2.778, 14.22)}$$