03-23-26 MTH 263

3.5 Related Rates
page 93: 1, 3, 6, 10

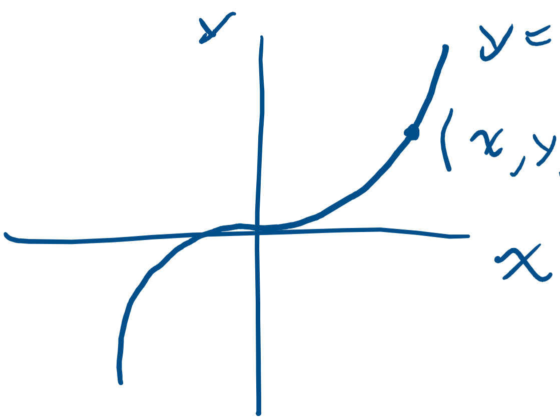
3.6 Differentials
page 98: 1, 3, 5, 9

Exam 2		stem & leaf		
51.45455	mean			A-0
23.3125	st.dev			B - 2
50	median	8	56	C - 0
11	min	7		D -3
86	max	6	114	F - 6
11	count	5	0	
		4	38	
		3		
		2	89	
		1	1	

Exam 1		stem & leaf		
56.16667	mean			A-0
20.88424	st.dev			B - 2
59	median	8	48	C - 0
21	min	7		D -3
88	max	6	299	F - 7
12	count	5	399	
		4	8	
		3	6	
		2	16	

3.5 : 6

6. An object moves along the curve $y = x^3$ in the xy-plane. At what points on the curve are the x and y coordinates of the object changing at the same rate?



$y = x^3$

$(x, y) = (x, x^3)$

Find (x, y) such that $\frac{dx}{dt} = \frac{dy}{dt}$

$\frac{dx}{dt} = \frac{d(x^3)}{dt}$

$\frac{dx}{dt} = 3x^2 \frac{dx}{dt}$

or

$\Rightarrow dx = dy$

$dx = d(x^3)$

$dx = 3x^2\, dx$

$$\frac{dx}{dt} = 3x^2 \frac{dx}{dt}$$

$$(3x^2 - 1)\frac{dx}{dt} = 0$$

$$3x^2 - 1 = 0 \text{ or } \frac{dx}{dt} = 0$$

$$3x^2 = 1$$

$$x^2 = \frac{1}{3}$$

$$\boxed{x = \pm\frac{1}{\sqrt{3}}}$$

$$y = \left(\pm\frac{1}{\sqrt{3}}\right)^3 = \boxed{\pm\frac{1}{3\sqrt{3}} = y}$$

$$dx = 3x^2\, dx$$

$$3x^2 = 1$$

$$x^2 = \frac{1}{3}$$

$$\boxed{x = \pm\frac{1}{\sqrt{3}}}$$

$\frac{dx}{dt} = 0 \Rightarrow$ point (x, y) does not move

$\Rightarrow \frac{dy}{dt} = 0$

3.5: 10

10. An angle θ changes with time. For what values of θ do $\sin\theta$ and $\tan\theta$ change at the same rate?

$$\frac{d\sin\theta}{dt} = \frac{d\tan\theta}{dt}$$

$$(\cos\theta)\frac{d\theta}{dt} = (\sec^2\theta)\frac{d\theta}{dt}$$

θ changes with time $\Rightarrow \frac{d\theta}{dt} \neq 0$

θ changes with time $\Rightarrow \frac{d\theta}{dt} \neq 0$

$\Rightarrow \cos\theta = \sec^2\theta$

$\cos\theta = \frac{1}{\cos^2\theta}$

$\cos^3\theta = 1$

$\boxed{\cos\theta = 1}$

$\boxed{\theta = 2\pi k, \quad k \in \mathbb{Z}}$

3.6

Memorize

For a differentiable function $f(x)$, the **differential** of $f(x)$ is

$$df = f'(x)\,dx \tag{3.7}$$

where dx is an infinitesimal change in x.

Example 3.32

Find the differential df of $f(x) = x^3$.

Solution: By definition,

$$df = f'(x)\,dx = 3x^2\,dx$$

Equivalently, this can be written as

$$d(x^3) = 3x^2\,dx\,,$$

which is often the way it would appear in textbooks in the sciences.

$\frac{df}{dx} = 3x^2$

$$\frac{df}{dx} = 3x^2$$

$$\Rightarrow df = 3x^2\,dx$$

Memorize

Let f and g be differentiable functions, and let c be a constant. Then:

(a) $d(c) = 0$

(b) $d(cf) = c\,df$ (Constant Multiple Rule)

(c) $d(f+g) = df + dg$ (Sum Rule)

(d) $d(f-g) = df - dg$ (Difference Rule)

(e) $d(fg) = f\,dg + g\,df$ (Product Rule)

(f) $d\left(\frac{f}{g}\right) = \frac{g\,df - f\,dg}{g^2}$ (Quotient Rule)

(g) $d(f^n) = nf^{n-1}\,df$ (Power Rule)

(h) $d(f(g)) = \frac{df}{dg}\,dg$ (Chain Rule)

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\,g'(x)$$

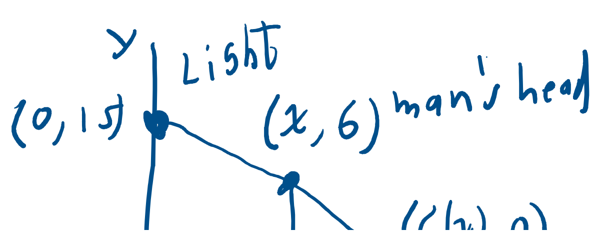
$$= \frac{df}{dg}(g(x))\,\frac{dg}{dx}$$

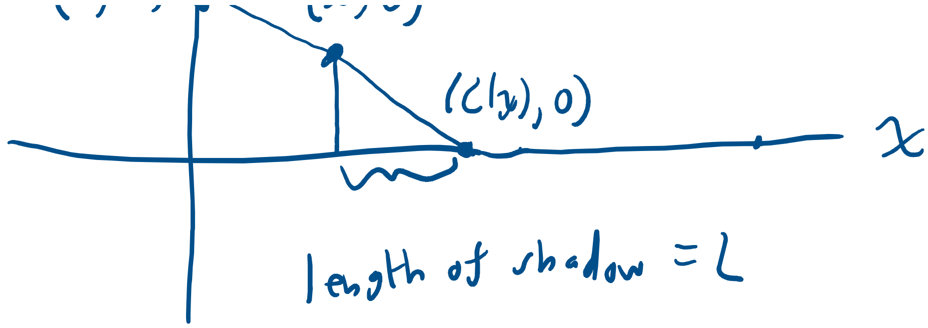
Multiply each side by dx

$$d f(g(x)) = \frac{df}{dg}(g(x))\,dg$$

3.5: 5

5. A person 6 ft tall is walking at a rate of 6 ft/s away from a light which is 15 ft above the ground. At what rate is the end of the person's shadow moving along the ground away from the light?





Let x = horizontal distance between man and light

given $\frac{dx}{dt} = 6 \frac{ft}{sec}$

Find $\frac{dL}{dt}$

$$\boxed{L = c(x) - x}$$

We have similar triangles.

$$\frac{c(x)}{15} = \frac{L}{6}$$

$$L = \frac{6\, c(x)}{15}$$

$$\boxed{L = \frac{2\, c(x)}{5}}$$

$$c(x) - x = \frac{2(c(x))}{5}$$

$$c(x) - \left(\frac{2}{5}\right) c(x) = x$$

$$c(x)\left(1 - \frac{2}{5}\right) = x$$

$$c(x)\left(\frac{3}{5}\right) = x$$

$$c(x) + \frac{5}{}$$

$$\boxed{c(x) = \frac{5x}{3}}$$

$$L = \frac{5x}{3} - x$$

$$\boxed{L = \frac{2x}{3}}$$

$$\frac{dL}{dt} = \frac{d}{dt}\left(\frac{2x}{3}\right)$$

$$= \frac{2}{3}\frac{d}{dt}(x)$$

$$\frac{dL}{dt} = \left(\frac{2}{3}\right)(6)\ \frac{ft}{sec}$$

$$\boxed{\frac{dL}{dt} = \frac{4\ ft}{sec}}$$