Took exam 2

3.5 Related Rates

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Related Rates: Expanding Sphere Problem

Please give me a related rates problem for first semester calculus. Do not give a hint or solution.

Related Rates: The Expanding Sphere

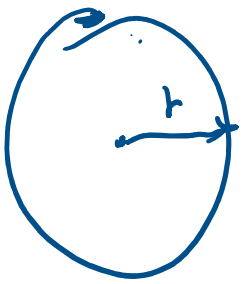
A weather balloon is being inflated with helium at a constant rate of **15 cubic feet per minute**.

At the exact instant when the radius of the balloon is **4 feet**, at what rate is the surface area of the balloon increasing?

Assume the balloon is a sphere

Let $A(r)$ = surface area $= 4\pi r^2$ (supplied)

r = radius $[ft^2]$



Let $V(r) = \left(\frac{4}{3}\right)\pi r^3$ [supplied]

Let t = time (min)

$\frac{dV}{dt} = 15 \frac{ft^3}{min}$

Find $\frac{dA}{dt}$ when $r = 4$ ft

$$\frac{dA}{dt}(r(t)) = \frac{dA}{dr} \cdot \frac{dr}{dt}$$

$$\frac{d}{dt}(\quad) = \frac{d}{dr}\cdot\frac{dr}{dt}$$

$$= \frac{d}{dr}\left(4\pi r^2\right)\frac{dr}{dt}$$

$f(a)$

$f|_{x=a}$

$$\boxed{\frac{dA}{dt} = 8\pi r\,\frac{dr}{dt}}$$

$$\boxed{\left.\frac{dA}{dt}\right|_{r=4ft} = 32\pi\,\frac{dr}{dt}\ ft}$$

$$\frac{dV}{dt} = \frac{dV}{dr}\cdot\frac{dr}{dt} = 15\ \frac{ft^3}{min}$$

$$\frac{dV}{dr} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2$$

$$\frac{dV}{dt} = \left(4\pi r^2\right)\frac{dr}{dt} = 15\ \frac{ft^3}{min}$$

$$\frac{dr}{dt} = \frac{15}{4\pi r^2}\ \frac{ft^3}{min}$$

$$\frac{dr}{dt} = \frac{15}{4\pi(4ft)^2}\ \frac{ft^3}{min}$$

$$\boxed{\frac{dr}{dt} = \frac{15}{16\pi}\ \frac{ft}{min}}$$

$$\frac{dA}{dt} = \left.\frac{dA}{dr}\right|_{r=15}\cdot\frac{dr}{dt}$$

$$\frac{dA}{dt} = (32\pi\ ft)\left(\frac{15}{16\pi}\right)\frac{ft}{min}$$

$$\boxed{\frac{dA}{dt} = 30\ \frac{ft^2}{min}}$$