03-09-26 MTH 263

3.4 Implicit Differentiation
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Exam 2, Wednesday, 03/11/26
2.4, 3.1 - 3.4

3.4:13

**13.** Find $\frac{d^2y}{dx^2}$ for the curve $x^2 + y^2 = 1$. You may use the results from Example 3.28.

from class of 03/04/26, we found

$$\boxed{\frac{dy}{dx} = -\frac{x}{y}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(-\frac{x}{y}\right)$$

$$= \frac{y\frac{d}{dx}(-x) - (-x)\frac{dy}{dx}}{y^2}$$

$$= \frac{-y + x\frac{dy}{dx}}{y^2}$$

$$= \frac{-y + x\left(-\frac{x}{y}\right)}{y^2}$$

$$= \frac{-y - \frac{x^2}{y}}{y^2}$$

$$= \boxed{\frac{-y^2 - x^2}{y^3}} = \frac{-(x^2 + y^2)}{y^3}$$

$y$ $\qquad$ $y$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-1}{y^3}}$$

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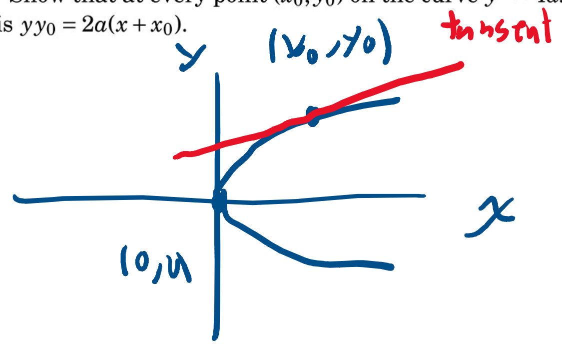
**14.** Show that at every point $(x_0, y_0)$ on the curve $y^2 = 4ax$, the equation of the tangent line to the curve is $yy_0 = 2a(x + x_0)$.




Let $a > 0$ (for the graph)

$\boxed{\text{w/o g} \quad \text{without loss of generality}}$

hint: point-slope equation of line

$$y - y_0 = f'(x_0, y_0)(x - x_0)$$

$$y^2 = 4ax$$

$$2y \frac{dy}{dx} = 4a$$

$$\boxed{\frac{dy}{dx} = \frac{2a}{y}}$$

$$y - y_0 = \frac{2a}{y_0}(x - x_0)$$

$$y = \frac{2a}{y_0}(x - x_0) + y_0$$

$$yy_0 = 2a(x - x_0) + y_0^2$$

$$yy_0 = 2a(x - x_0) + 4ax_0$$

$yy_0 = 2a(x - x_0) + 4ax_0$

$yy_0 = 2ax - 2ax_0 + 4ax_0$

$yy_0 = 2ax + 2ax_0$

$\boxed{yy_0 = 2a(x + x_0)}$

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**Squeeze Theorem**: Suppose that for some functions $f$, $g$ and $h$ there is a number $x_0 \geq 0$ such that

$$g(x) \leq f(x) \leq h(x) \text{ for all } x > x_0$$

and that $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} h(x) = L$. Then $\lim_{x\to\infty} f(x) = L$.

Similarly, if $g(x) \leq f(x) \leq h(x)$ for all $x \neq a$ in some interval $I$ containing $a$, and if $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} f(x) = L$.

Suppose we are given a complicated function and asked to find its limit.
What do we need in order to apply the squeeze theorem?
Is there an algorithm for this?

To apply the squeeze theorem, we need functions g(x) and h(x) that satisfy the conditions of the theorem.
No, I don't know of any general algorithm for this.

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Let $f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}$

Is $f(x)$ continuous at $x = 0$?

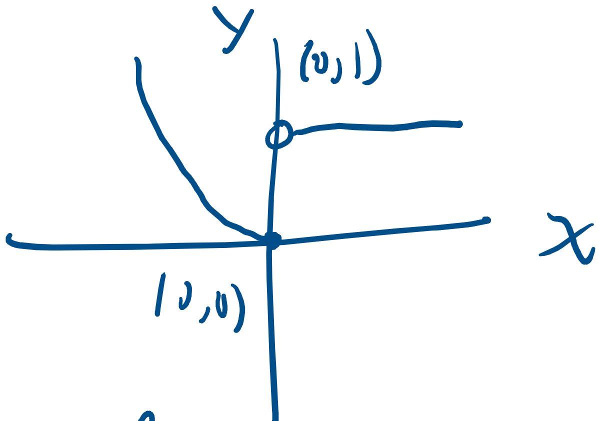
Use the definition of continuity.

TI $\quad Y_1 = x^2(x \leq 0) + 1(x > 0)$

TI $\quad Y_1 = x^2(x \leq 0) + 1(x > 0)$

Def $f(x)$ is continuous at $x = c$

if $\lim_{x \to c} f(x) = f(c)$




$\lim_{x \to 0^-} f(x) = 0$

$\lim_{x \to 0^+} f(x) = 1$

$0 \neq 1$

$\therefore f(x)$ is not cont. at $x = 0$

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show $\lim_{x \to a} x = a$

Let $\varepsilon > 0$

Find $\delta > 0$ such that

$0 < |x - a| < \delta \Rightarrow |x - a| < \varepsilon$

Let $\delta = \varepsilon$

$-\delta < x - a < \delta$

check example on calculator

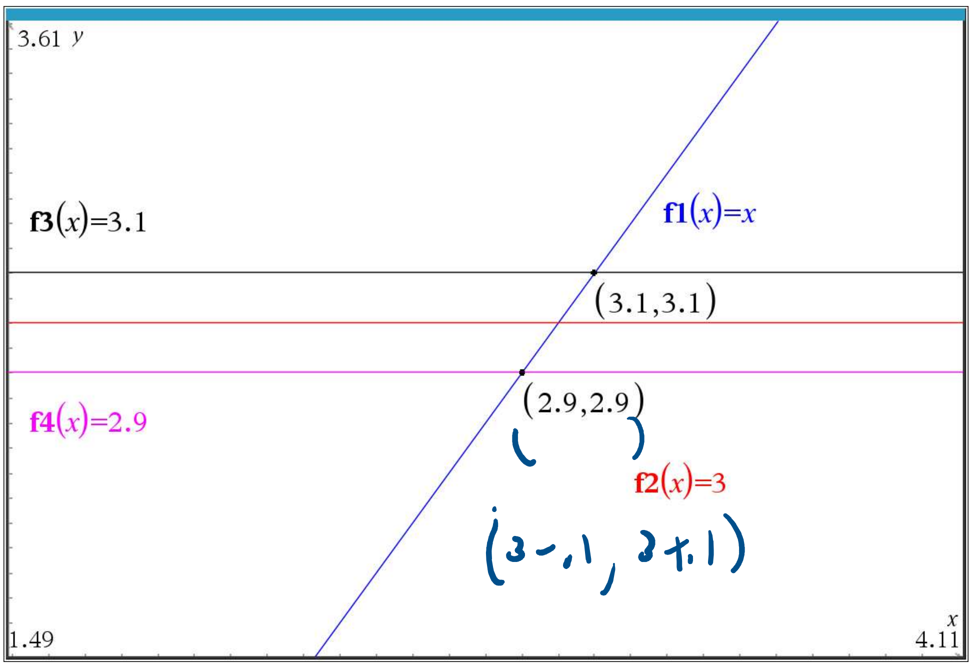
[illegible] example [illegible]

Find $\lim_{x \to 3} x$

$\lim_{x \to 3} x = 3$

Let $\varepsilon = 0.1$

Find $\delta$ that satisfies formal definition of limit




$3.1 - 3 = 0.1$

$3 - 2.9 = 0.1$

$\therefore \delta = 0.1 = \varepsilon$

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No floor or ceiling function on exam 2