03-02-26 MTH 263

3.2 Limits: Formal Definition
page 82: 1, 3, 5, 7, 13
3.3 Continuity
page 88: 1, 5, 15, 25, 29

12*1.0385=12.462 sections to be on track. We are at 13

3.2

A

For Exercises 1-18 evaluate the given limit.

1. $\lim\limits_{x\to 2} \dfrac{x^2+3x-10}{x^2-x-2}$ $\dfrac{2^2+(3)(2)-10}{2^2-2-2} = \dfrac{4+6-10}{4-2-2} = \dfrac{0}{0}$ not defined

$\left[\frac{0}{0}\right]$

$\overset{=}{L} \lim\limits_{x\to 2} \dfrac{\frac{d}{dx}(x^2+3x-10)}{\frac{d}{dx}(x^2-x-2)} = \lim\limits_{x\to 2}\dfrac{(2x+3)}{(2x-1)}$

$= \dfrac{2\cdot(2)+3}{2(2)-1} = \boxed{\dfrac{7}{3}}$

3. $\lim\limits_{x\to\infty} \dfrac{x^2+3x-10}{2x^3-x-2}$ $\left[\frac{\infty}{\infty}\right]$

From precalculus, numerator deg < denom deg

$\Rightarrow$ horizontal asymptote $\boxed{y=0}$

$\overset{=}{L} \lim\limits_{x\to\infty} \dfrac{\frac{d}{dx}(x^2+3x-10)}{\frac{d}{dx}(2x^3-x-2)} = \lim\limits_{x\to\infty}\dfrac{2x+3}{6x^2-1}$ $\left[\frac{\infty}{\infty}\right]$

$\overset{=}{L} \lim\limits_{x\to\infty} \dfrac{\frac{d}{dx}(2x+3)}{\frac{d}{dx}(6x^2-1)} = \lim\limits_{x\to\infty}\dfrac{2}{12x} = \boxed{0}$

3.2: 5

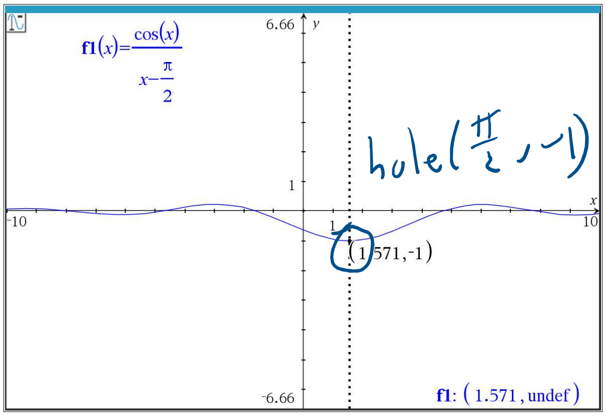
For Exercises 1-18 evaluate the given limit.

5. $\lim\limits_{x\to\pi/2} \dfrac{\cos x}{x-\pi/2}$ $\left[\frac{0}{0}\right]$ $\overset{=}{L} \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\frac{d}{dx}(\cos x)}{\frac{d}{dx}\left(x-\frac{\pi}{2}\right)}$

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{(-\sin x)}{1} = \lim\limits_{x\to\frac{\pi}{2}}(-\sin x)$

$x \to \frac{\pi}{2}$

$= -\sin\left(\frac{\pi}{2}\right) = \boxed{-1}$



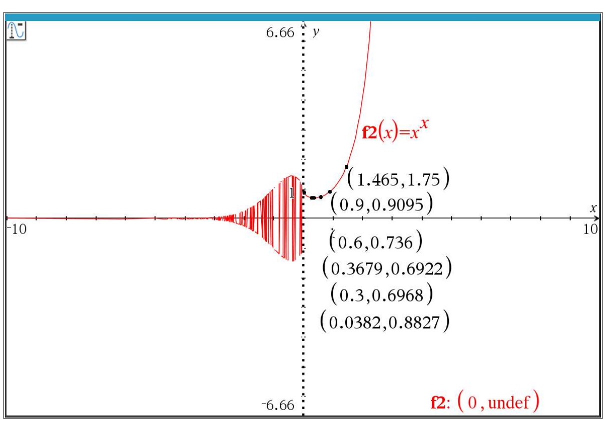
3.3
Memorize

If f is a continuous function and $\lim_{x\to a} g(x)$ exists and is finite, then:

$$f\left(\lim_{x\to a} g(x)\right) = \lim_{x\to a} f(g(x)) \tag{3.5}$$

The same relation holds for one-sided limits.

$$x = \frac{1}{\frac{1}{x}} = \frac{\frac{1}{1}}{\frac{1}{x}} = \frac{1}{1}\cdot\frac{x}{1} = x$$



Memorize theorem

Every differentiable function is continuous.

Proof: If a function f is differentiable at $x=a$ then $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exists, so

$$\lim_{x\to a}(f(x)-f(a)) = \lim_{x\to a}(f(x)-f(a))\cdot\frac{x-a}{x-a} = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}\cdot\lim_{x\to a}(x-a) = f'(a)\cdot 0 = 0$$

which means that $\lim_{x\to a} f(x) = f(a)$, i.e. f is continuous at $x=a$. ✓

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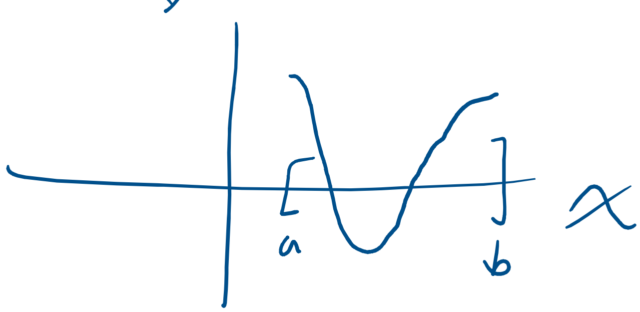
Memorize

Extreme Value Theorem: If f is a continuous function on a closed interval $[a,b]$ then f attains both a maximum value and a minimum value on that interval.

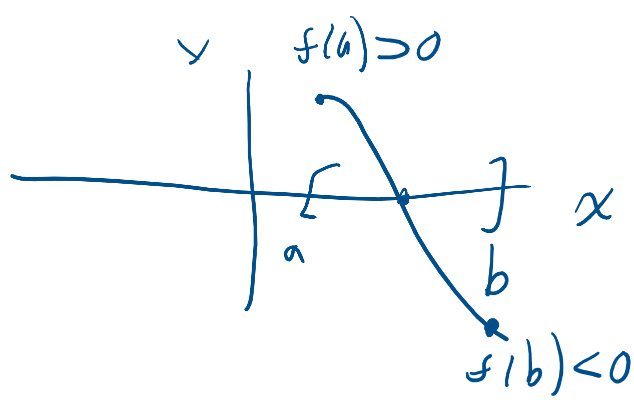
[a, b] is a bounded interval



memorize

Intermediate Value Theorem: If f is a continuous function on a closed interval $[a,b]$ then f attains every value between $f(a)$ and $f(b)$.

Special case: Let f(x) be continuous on [a, b] and let f(a) and f(b) have opposite signs.
Then there exists a zero in the open interval (a, b)



3.3

For Exercises 1-18, indicate whether the given function $f(x)$ is continuous or discontinuous at the given value $x=a$ by comparing $f(a)$ with $\lim_{x\to a} f(x)$.

2. $f(x)=|x-1|$; at $x=0$

2. $f(x) = |x - 1|$; at $x = 0$

$f(0) = |0 - 1| = |-1| = \boxed{1}$

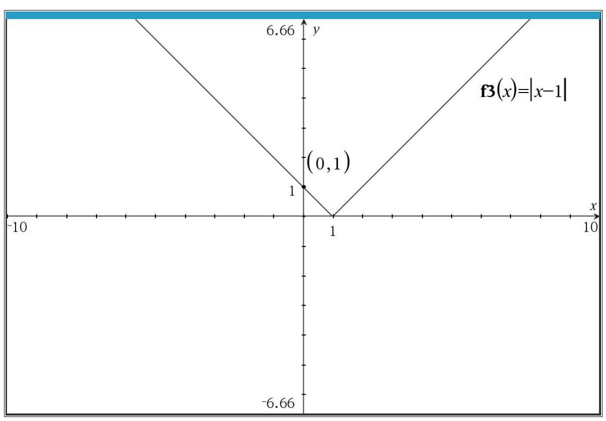
$\lim_{x \to 0} |x-1| = ?$

$|x-1| = \begin{cases} x-1 & \text{if } x-1 \geq 0 \quad \text{if } x \geq 1 \\ -x+1 & \text{if } x-1 < 0 \quad \text{if } x < 1 \end{cases}$

For x "close" to 0, $x - 1 < 0$

$\lim_{x \to 0} |x-1| = \lim_{x \to 0} (-x+1) = -0 + 1 = \boxed{1}$

$\therefore f$ is cont. at $x = 0$



For Exercises 1-18, indicate whether the given function $f(x)$ is continuous or discontinuous at the given value $x = a$ by comparing $f(a)$ with $\lim_{x \to a} f(x)$.

10. $f(x) = \begin{cases} 0 & \text{if } x \leq 0, \\ 1 & \text{if } x > 0; \end{cases}$

at $x = 0$

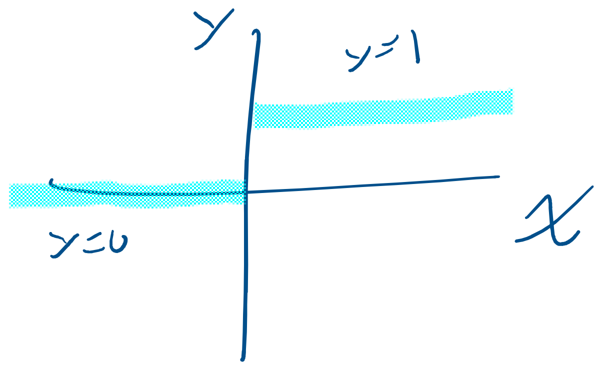
$\boxed{f(0) = 0}$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(0) = \boxed{0}$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(1) = \boxed{1}$$

$$0 \neq 1$$

$\therefore$ f is not at $x = 0$



Your Name MTH 263 quiz 3 calculator needed.
Closed book, closed notes

1. Use calculus to find the equation of the tangent line to $f(x) = 2x^3 - 3$ at $x = 1$.

$y - y_0 = m(x - x_0)$

$x_0 = 1$

$f(1) = 2(1^3) - 3 = \boxed{-1}$

point of tangency $= (1, -1)$

$m = f'(1)$

$f'(x) = 6x^2$

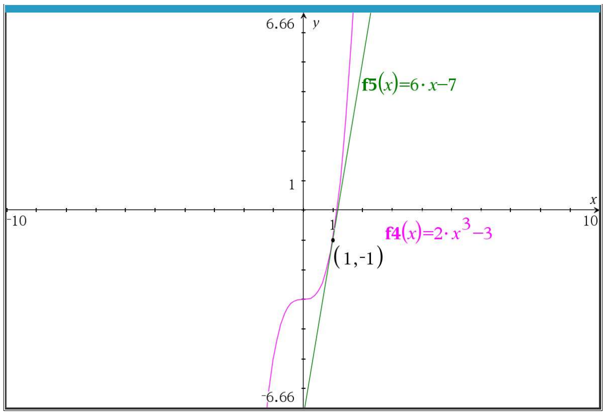
$f'(1) = \boxed{6}$

$y - (-1) = 6(x - 1)$

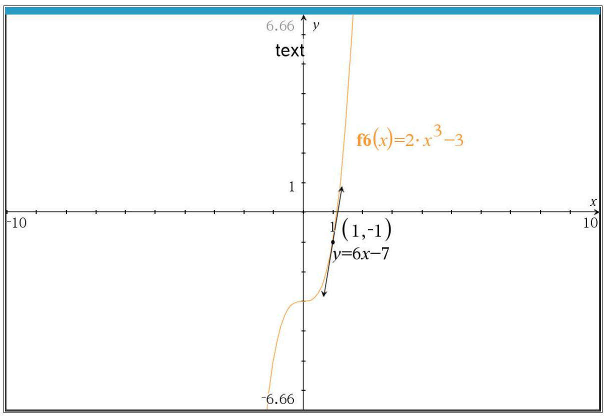
$y = -1 + 6x - 6$

$\boxed{y = 6x - 7}$

2. Verify your answer to #1 on your calculator.
Explain your procedure and sketch a labeled graph.
I graphed f(x) and y = 6x-7. Then, I found the intersection point to be (1,-1).



I graphed f(x). Then, I had TI draw the tangent line at x = 1. It gave the equation of the tangent line: y = 6x-7.



3. Evaluate the given limit.

$$\lim_{x\to 0} \frac{\ln(1-x)\ -\ \sin^2 x}{1\ -\ \cos^2 x} \qquad \boxed{\frac{0}{0}}$$

Sketch a labeled graph from your calculator, using Trace to plot a couple of nearby points to estimate the limit.

$$\overset{L}{=} \lim_{x\to 0} \frac{\frac{d}{dx}\left(\ln(1-x) - \sin^2 x\right)}{\frac{d}{dx}\left(1 - \cos^2 x\right)}$$

$$= \lim_{x\to 0} \frac{\left(\frac{1}{1-x}\right)(-1) - 2\sin x\cos x}{-2\cos x\sin x}$$

$$= \lim_{x\to 0} \left(\frac{-1}{(1-x)(-2\cos x\sin x)} - 1\right)$$

$$= \lim_{x\to 0} \left(\frac{1}{(1-x)\sin 2x} + 1\right)$$

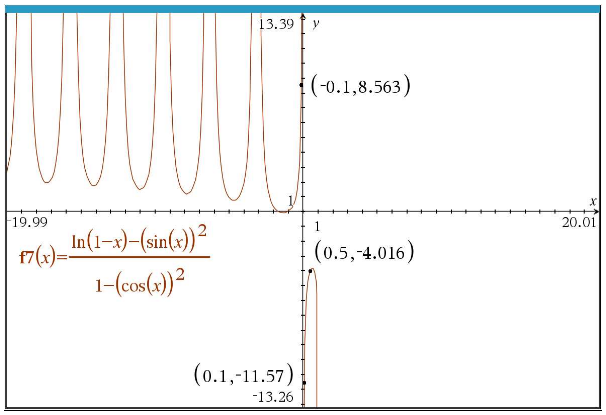
$$= \lim_{x\to 0} \left(\frac{1}{(1-x)\sin 2x}\right) + 1$$

$$\lim_{x\to 0^-} \left(\frac{1}{(1-x)(\sin 2x)}\right) = \infty\ (dne)$$

$$\lim_{x\to 0^+} \left(\frac{1}{(1-x)(\sin 2x)}\right) = -\infty\ (dne)$$

$$\lim_{x \to 0^+} \left(\frac{1}{(1-x)(\sin 2x)} \right) = -\infty \text{ (dne)}$$

$$\therefore \lim_{x \to 0} f(x) \text{ dne}$$



4. Write the formal definition of $\lim_{x \to a} f(x) = L$.

$\forall \varepsilon > 0, \exists \delta > 0$ such that

$0 < |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$