02-18-26 MTH 263

2.4 General Exponential and Logarithmic Functions

page 65: 1, 3, 5, 7

3 Topics in Differential Calculus

3.1 Tangent Lines

page 71: 1, 5, 11, 15, 23, 27

2.4 memorize or be able to derive

$$\frac{d}{dx}\left(a^x\right) = (\ln a)\, a^x$$

$$\frac{d}{dx}\left(a^u\right) = (\ln a)\, a^u \cdot \frac{du}{dx}$$

$$a^x = e^{x \ln a}$$

Memorize (this is from precalculus)

$$\log_a(bc) = \log_a b + \log_a c \qquad a^b \cdot a^c = a^{b+c}$$

$$\log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c \qquad \frac{a^b}{a^c} = a^{b-c}$$

$$\log_a b^c = c \log_a b \qquad \left(a^b\right)^c = a^{bc}$$

$$\log_a 1 = 0 \qquad a^0 = 1$$

Memorize (you need this for a TI-83)

$$\log_a x = \frac{\ln x}{\ln a}$$

Memorize or be able to derive

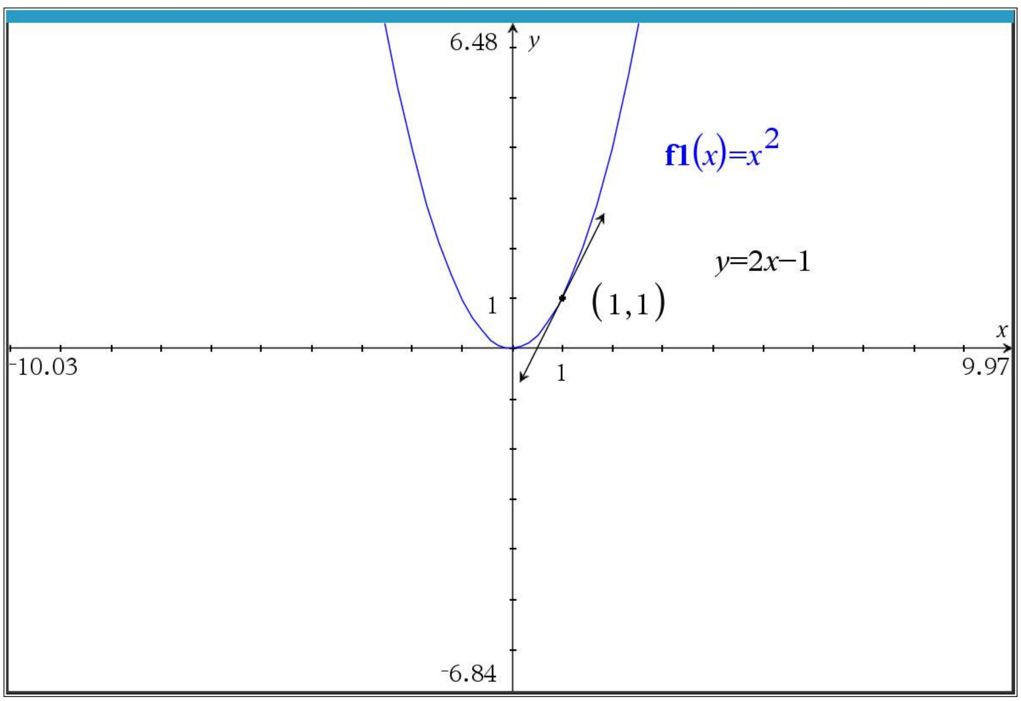
$$\frac{d}{dx}\left(\log_a x\right) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}\left(\log_a u\right) = \frac{1}{u \ln a} \cdot \frac{du}{dx} = \frac{u'}{u \ln a}$$

3.1 memorize

For a curve $y = f(x)$ that is differentiable at $x = a$, the **tangent line** to the curve at the point $P = (a, f(a))$ is the unique line through P with slope $m = f'(a)$. P is called the **point of tangency**. The equation of the tangent line is thus given by:

$$y - f(a) = f'(a)\cdot(x - a) \tag{3.1}$$



Understand and remember

- *The tangent line can be thought of as a limit of secant lines.*
  A **secant line** to a curve is a line that passes through two points on the curve. Figure 3.1.5 shows a secant line L_{PQ} passing through the points $P = (x_0, f(x_0))$ and $Q = (w, f(w))$ on the curve $y = f(x)$,

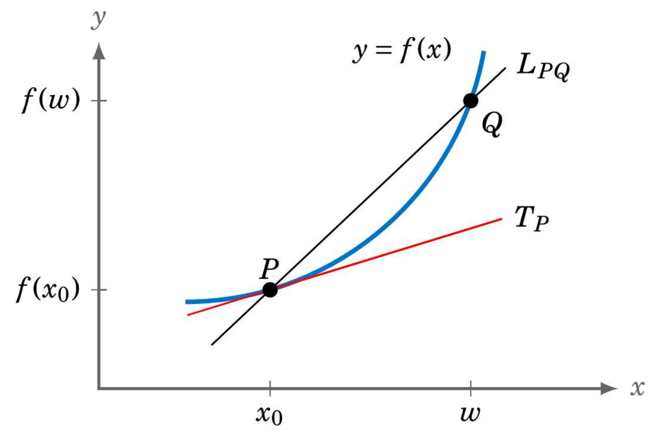


Figure 3.1.5 Secant line L_{PQ} approaching the tangent line T_P as $Q \to P$