02-11-26 MTH 263

2 Derivatives of Common Functions

2.1 Inverse Functions

page 50: 1, 3, 5

2.2 Trigonometric Functions and Their Inverses

page 54: 1, 4, 5, 15

2.3 The Exponential and Natural Logarithm Functions

page 62: 1, 5, 7, 9, 13, 17, 19

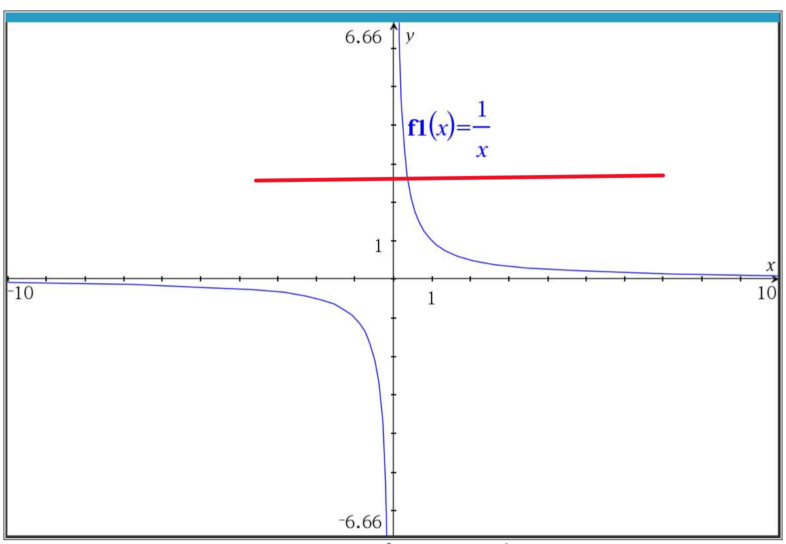
Exam 1 - Wednesday - 02/18/26; 1.1-1.6, 2.1-2.3

2.1: 5

A

For Exercises 1-8, show that the given function $y = f(x)$ is one-to-one over the given interval, then find the formulas for the inverse function f^{-1} and its derivative. Use Example 2.2 as a guide, including putting f^{-1} and its derivative in terms of x.

5. $f(x) = \frac{1}{x}$, for all $x > 0$



show $f(x)$ is 1-1

Let $f(c) = f(d)$

show $c = d$

$\frac{1}{c} = \frac{1}{d}$ Assume $c \neq 0$, $d \neq 0$

$\overline{c} = \overline{d}$ Assume $c \neq$ $d \neq 0$

$\Rightarrow c = d$ ✓

$\therefore f$ is 1-1

Example 2.2

Find the inverse f^{-1} of the function $f(x) = x^3$ then find the derivative of f^{-1}.

Solution: Rewrite $y = f(x) = x^3$ as $x = f(y) = y^3$, so that its inverse function $y = f^{-1}(x) = \sqrt[3]{x}$ has derivative

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{3y^2} \quad \text{, which is in terms of } y \text{, so putting it in terms of } x \text{ yields}$$

$$= \frac{1}{3\left(\sqrt[3]{x}\right)^2} = \frac{1}{3x^{2/3}}$$

which agrees with the derivative obtained by differentiating $y = \sqrt[3]{x}$ directly.

$y = f(x) = \frac{1}{x}$ switch x, y

$x = f(y) = \frac{1}{y}$

$f^{-1}(x) = f^{-1}(f(y)) = \frac{1}{y}$

$f^{-1}(x) = y$

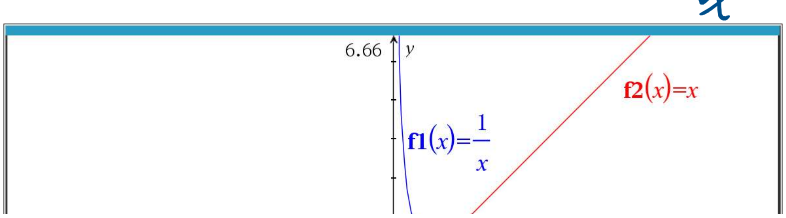
$y = \frac{1}{x}$

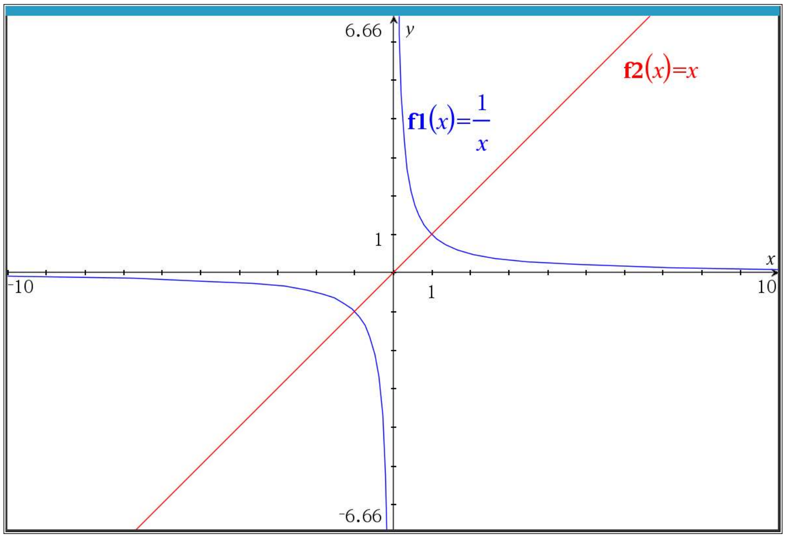
$x = \frac{1}{y}$

$y = \frac{1}{x}$

$\boxed{f^{-1}(x) = \frac{1}{x}}$

check $f(f^{-1}(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x$ ✓





$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\frac{d\left(\frac{1}{y}\right)}{dy}}$$

$$\frac{dy}{dx} = \frac{1}{\frac{d}{dy}\left(y^{-1}\right)} = \frac{1}{-y^{-2}} = -\frac{1}{\frac{1}{y^2}} = -y^2$$

$$\frac{dy}{dx} = -y^2$$

$$\frac{dy}{dx} = -\left(\frac{1}{x}\right)^2$$

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

$$\boxed{\frac{d}{dx}\left(\frac{1}{x}\right)^{-1} = -\frac{1}{x^2}}$$

2.2: 5

For Exercises 1-16, find the derivative of the given function $y = f(x)$.

5. $y = \tan^{-1}(x/3)$

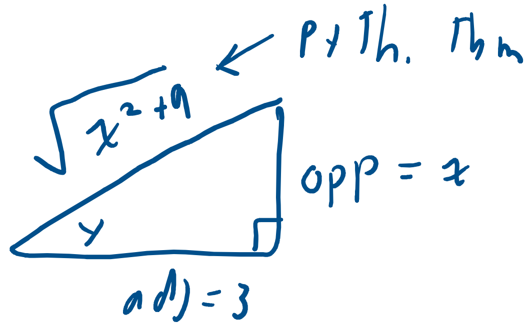
$$\tan y = \frac{x}{3}$$

$$\frac{d}{dx}(\tan y) = \frac{d}{dx}\left(\frac{x}{3}\right)$$

$$(\sec^2 y)\frac{dy}{dx} = \frac{1}{3}$$

$$\frac{dy}{dx} = \frac{1}{3\sec^2 y}$$

$$\frac{dy}{dx} = \frac{\cos^2 y}{3}$$



$$\cos y = \frac{adj}{hyp} = \frac{3}{\sqrt{x^2+9}}$$

$$\frac{dy}{dx} = \frac{9}{3(x^2+9)}$$

$$\boxed{\frac{dy}{dx} = \frac{3}{x^2+9}}$$

$$\boxed{\frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{3}\right)\right) = \frac{3}{x^2+9}}$$

$$\frac{dy}{dx} = \frac{3}{x^2+9}$$

$$\frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{3}\right)\right) = \frac{3}{x^2+9}$$

$$\frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{3}\right)\right) \qquad \frac{3}{x^2+9}$$

2.3
Memorize

$$\ln(ab) = \ln a + \ln b \qquad e^a \cdot e^b = e^{a+b}$$

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b \qquad \frac{e^a}{e^b} = e^{a-b}$$

$$\ln a^b = b \ln a \qquad (e^a)^b = e^{ab}$$

$$\ln 1 = 0 \qquad e^0 = 1$$

$$e \approx \left(1+\frac{1}{x}\right)^x \quad \Rightarrow \quad e^{1/x} \approx \left(\left(1+\frac{1}{x}\right)^x\right)^{1/x} = 1 + \frac{1}{x} \quad \Rightarrow \quad \left(e^{1/x} - 1\right)x \approx \left(\frac{1}{x}\right)x = 1,$$

$$\Rightarrow e^{1/x} - 1 \approx \frac{1}{x}$$

$$\Rightarrow \left(e^{\frac{1}{x}} - 1\right)(x) \approx \left(\frac{1}{x}\right)(x)$$

$$\Rightarrow \left(e^{\frac{1}{x}} - 1\right)(x) \approx 1$$

let $h = \frac{1}{x}$

$$\left(e^{h} - 1\right)\left(\frac{1}{h}\right) \approx 1$$

$$\frac{e^h - 1}{h} \approx 1$$

$x \to \infty \Rightarrow h \to 0$

$$\boxed{\lim_{h \to 0} \frac{e^h - 1}{h} = 1}$$ supplied

Memorize

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}\left(e^u\right) = e^u \cdot \frac{du}{dx}$$

Memorize

domain of $\ln x$	$=$	all $x > 0$	$=$	range of e^x
range of $\ln x$	$=$	all x	$=$	domain of e^x

$$y = e^x \quad \text{if and only if} \quad x = \ln y \tag{2.3}$$

$$e^{\ln x} = x \quad \text{for all } x > 0 \tag{2.4}$$

$$\ln\left(e^x\right) = x \quad \text{for all } x \tag{2.5}$$

Memorize

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Memorize

$$\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx} = \frac{u'}{u}$$

Memorize

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$

Your Name MTH 263 quiz 2 write each problem. Calculator OK.

1. Write the formal definition of $\lim_{x\to c} f(x) = L$. ε - δ My mistake. We will learn this later.

2. Let $f(x) = (6x^2 - 5x)^4$. Find $\frac{df}{dx}$.

$$\frac{df}{dx} = 4(6x^2 - 5x)^3 (12x - 5)$$

3. Use

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \quad (\text{for } |x| < 1)$$

to find $\frac{dy}{dx}$ for

$$y = (\sin^{-1} 3x)^2$$

$$\frac{dy}{dx} = 2(\sin^{-1} 3x)\frac{1}{\sqrt{1-(3x)^2}}(3)$$

$$\boxed{\frac{dy}{dx} = \frac{6\sin^{-1}(3x)}{\sqrt{1-9x^2}}}$$

TI-nspire check

$$\frac{d}{dx}\left(\left(\sin^{-1}(3\cdot x)\right)^2\right) \qquad \frac{6\cdot \sin^{-1}(3\cdot x)}{\sqrt{1-9\cdot x^2}}$$

4.	Let $f(x) = (8x^2 + 2)(x^3 - 1)$.

Find $f'(1)$. You can check with your calculator, but manual calculation required for credit.

$$f'(x) = (8x^2+2)(3x^2) + (x^3-1)(16x)$$

$$= 24x^4 + 6x^2 + 16x^4 - 16x$$

$$f'(x) = 40x^4 + 6x^2 - 16x$$

$$f'(1) = 40(1) + 6(1) - 16 = 46 - 16 = \boxed{30}$$

5. Calculate $\lim_{x \to -1} \frac{x^2+2x+1}{x+1}$.

$$= \lim_{x \to -1} \frac{(x+1)^2}{x+1} = \lim_{x \to -1} (x+1) = -1+1 = \boxed{0}$$