01-26-26 MTH 263

1 The Derivative
- 1.1 Introduction
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- 1.2 The Derivative: Limit Approach
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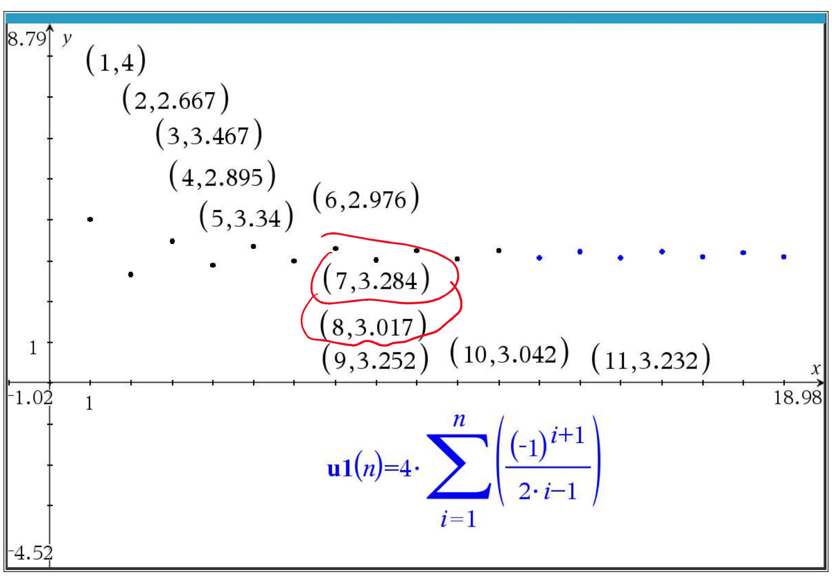
NOVA Code Red, so we held the class in Zoom.
The link to the Zoom recording will be posted in Canvas when I receive it.

1.1

5. By equation (1.1), $\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots\right)$, where the n^{th} term in the sum inside the parentheses is $\frac{(-1)^{n+1}}{2n-1}$ (starting at $n = 1$).[8] So the first approximation of π using this formula is $\pi \approx 4(1) = 4.0$, and the second approximation is $\pi \approx 4\left(1-\frac{1}{3}\right) = 8/3 \approx 2.66667$. Continue like this until two consecutive approximations have 3 as the first digit before the decimal point. How many terms in the sum did this require? Be careful with rounding off in the approximations.

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

summation notation



We needed 8 terms to have two consecutive 3's before the decimal point.

1.2

Memorize

The **derivative** of a real-valued function $f(x)$, denoted by $f'(x)$, is

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \tag{1.3}$$

for x in the domain of f, provided that the limit exists.[11]

Example 1.1

Find the derivative of the function $f(x) = 1$.

Solution: By definition, $f(x) = 1$ for all x, so:

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{1-1}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{0}{\Delta x} \\ &= \lim_{\Delta x \to 0} 0 \\ f'(x) &= 0 \end{aligned}$$

Memorize

The derivative of any constant function is 0.

Let $f(x) = c$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{c-c}{\Delta x} = \frac{0}{\Delta x} = 0$$

$$\lim_{\Delta x \to 0} 0 = \boxed{0} = f'(x)$$

Memorize

The derivative of any linear function is the slope of the line itself:
If $f(x) = mx + b$ then $f'(x) = m$ for all x.

Memorize

For a real number a and a real-valued function $f(x)$, say that the *limit* of $f(x)$ as x approaches a equals the number L, written as

$$\lim_{x \to a} f(x) = L\,,$$

if $f(x)$ approaches L as x approaches a.
Equivalently, this means that $f(x)$ can be made as close as you want to L by choosing x close enough to a. Note that x can approach a from any direction.

Memorize

Rules for Limits: Suppose that a is a real number and that $f(x)$ and $g(x)$ are real-valued functions such that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist. Then:

(a) $\lim_{x\to a} (f(x)+g(x)) = \left(\lim_{x\to a} f(x)\right) + \left(\lim_{x\to a} g(x)\right)$

(b) $\lim_{x\to a} (f(x)-g(x)) = \left(\lim_{x\to a} f(x)\right) - \left(\lim_{x\to a} g(x)\right)$

(c) $\lim_{x\to a} (k\cdot f(x)) = k\cdot\left(\lim_{x\to a} f(x)\right)$ for any constant k

(d) $\lim_{x\to a} (f(x)\cdot g(x)) = \left(\lim_{x\to a} f(x)\right)\cdot\left(\lim_{x\to a} g(x)\right)$

(e) $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, if $\lim_{x\to a} g(x) \neq 0$

Alternative notation for the derivative

$$\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}}$$

$(x+h) - x = x - x + h$
$= 0 + h = h$
$= \Delta x$

Alternate formulations for the derivative are given in the text.
They will be supplied if needed.

Supplied

The derivative of an even function is an odd function.
The derivative of an odd function is an even function.

Def $f(x)$ is even if $f(-x) = f(x)$
$f(x)$ is odd if $f(-x) = -f(x)$

Find the derivative, using the limit of the difference quotient.

$f(x) = 8x - 7$

$f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \dfrac{\Delta f}{\Delta x}$

$\dfrac{\Delta f}{} = \dfrac{f(x+h) - f(x)}{}$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{[8(x+h) - 7] - [8x - 7]}{h}$$

$$= \frac{\cancel{8x} + 8h \cancel{- 7} \cancel{- 8x} \cancel{+ 7}}{h}$$

$$= \frac{8\cancel{h}}{\cancel{h}}$$

$$\boxed{\frac{\Delta f}{\Delta x} = 8}$$

$$f'(x) = \lim_{h \to 0} (8) = \boxed{8}$$

Alternative presentation

$$f'(x) = \lim_{h \to 0} \frac{\Delta f}{\Delta x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[8(x+h) - 7] - (8x - 7]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{8x} + 8h \cancel{- 7} \cancel{- 8x} \cancel{+ 7}}{h}$$

$$= \lim_{h \to 0} \frac{8\cancel{h}}{\cancel{h}}$$

$$= \lim_{h\to 0} (8) = \boxed{8}$$

Find $f'(x)$ for $f(x) = \sqrt{x}$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \frac{(\sqrt{x+h} - \sqrt{x})}{h} \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{\cancel{x}+h - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{\cancel{h}\ 1}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$\boxed{\frac{\Delta f}{\Delta x} = \frac{1}{\sqrt{x+h} + \sqrt{x}}}$$

$$f'(x) = \lim_{h\to 0} \frac{\Delta f}{\Delta x} = \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}}$$

$\sqrt{x+0}+\sqrt{x} \quad \sqrt{x}+\sqrt{x}$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x}}}$$

Find $f'(x)$ for $f(x) = 3x^2 - 4x + 5$

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{\left[3(x+h)^2 - 4(x+h) + 5\right] - \left[3x^2 - 4x + 5\right]}{h}$$

$$= \frac{3(x^2 + 2xh + h^2) - 4x - 4h + 5 - 3x^2 + 4x - 5}{h}$$

$$= \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{- 4x} - 4h \cancel{+ 5} \cancel{- 3x^2} \cancel{+ 4x} \cancel{- 5}}{h}$$

$$= \frac{6xh + 3h^2 - 4h}{h}$$

$$= \frac{\cancel{h}(6x + 3h - 4)}{\cancel{h}}$$

$$\boxed{\frac{\Delta f}{\Delta x} = 6x + 3h - 4}$$

$$f'(x) = \lim_{h \to 0} \frac{\Delta f}{\Delta x} = \lim_{h \to 0} (6x + 3h - 4)$$

$$= 6x + 3(0) - 4$$

$$\boxed{f'(x) = 6x - 4}$$

$$\boxed{f'(x) = 6x - 4}$$

Find $\lim\limits_{x \to 3} (4x^2 - 5x)$

$$= \lim_{x \to 3} (4x^2) + \lim_{x \to 3} (-5x)$$

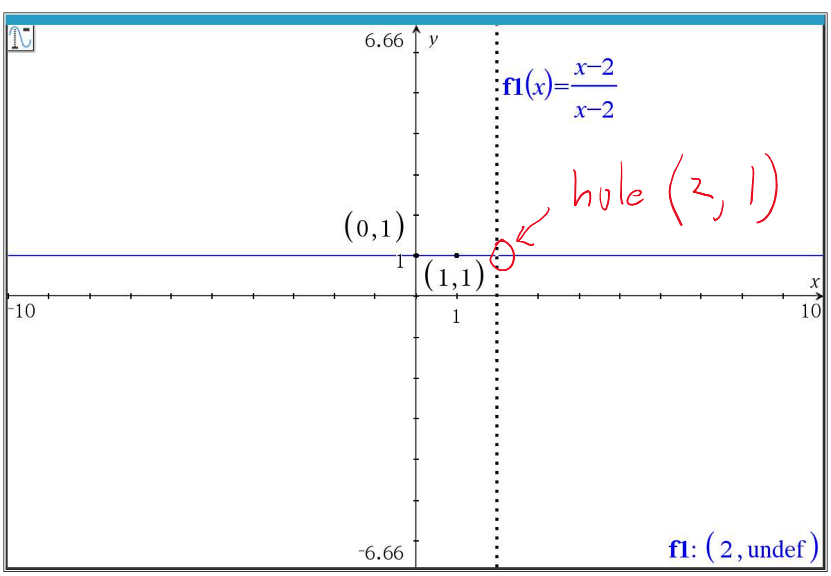
$$= 4 \lim_{x \to 3} (x^2) - 5 \lim_{x \to 3} (x)$$

$$= 4(3^2) - 5(3)$$

$$= 36 - 15$$

$$= \boxed{21}$$

$$\lim_{x \to 2} \frac{x-2}{x-2} = \frac{2-2}{2-2} = \frac{0}{0} \text{ not defined}$$



$$\lim_{x \to 2} \frac{x-2}{x-2} = \lim_{x \to 2} 1 = \boxed{1}$$

$$\lim_{x \to 3} \frac{2x - x^2}{\ln(x)} = \frac{2(3) - (3)^2}{\ln(3)}$$

$$= \frac{6-9}{\ln(3)} = \boxed{\frac{-3}{\ln(3)}}$$

TI check

$$\lim_{x \to 3} \left(\frac{2 \cdot x - x^2}{\ln(x)} \right) \qquad \frac{-3}{\ln(3)}$$