

14 textbook sections remaining
 12 class meetings before final exam
 $14/12=1.1667$

1-2 textbook sections per class

3.3 Continuity

page 88: 1, 5, 15, 25, 29

3.4 Implicit Differentiation

page 91: 1, 7, 13, 14

Exam 2		stem & leaf		
63.5	mean	9 07	A-2	
61.5	median	8 08	B-2	
21.6852	st. dev	7 7	C-1	
5				
30	min	6 7	D-1	
97	max	5 6	F-6	
12	count	4 389		
		3 07		

Exam 1		stem & leaf		
53.16667	mean		A-0	
47	median	8 58	B-2	
20.73979	st. dev	7 8	C-1	
20	min	6 3	D-1	
88	max	5 09	F- 8	
12	count	4 24		
		3 379		
		2 0		

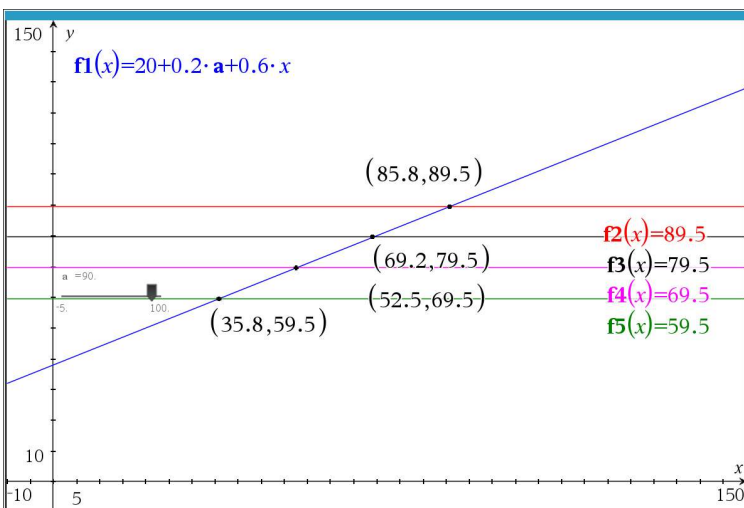
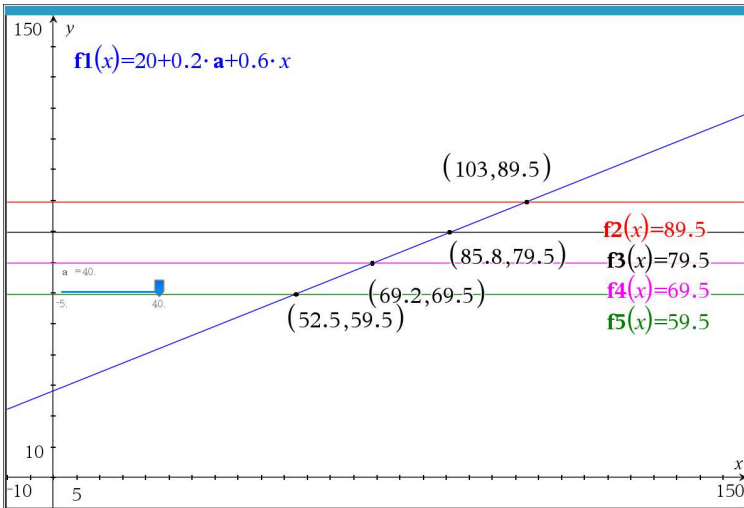
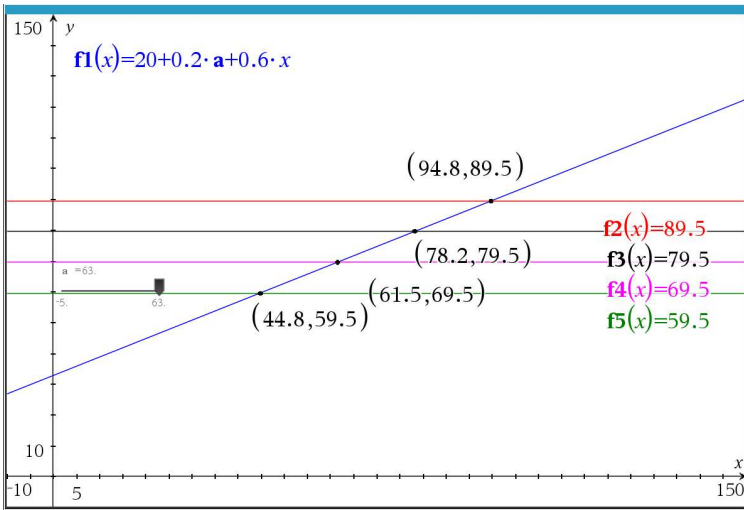
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3.3 Memorize

A function f is **continuous** at $x = a$ if

$$\lim_{x \rightarrow a} f(x) = f(a). \quad (3.4)$$

A function is continuous on an interval I if it is continuous at every point in the interval. For a closed interval $I = [a, b]$, a function f is continuous on I if it is continuous on the open interval (a, b) and if $\lim_{x \rightarrow a^+} f(x) = f(a)$ (i.e. f is **right continuous** at $x = a$) and $\lim_{x \rightarrow b^-} f(x) = f(b)$ (i.e. f is **left continuous** at $x = b$). A function is **discontinuous** at a point if it is not continuous there. A continuous function is one that is continuous over its entire domain.

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$$

and $f(a)$ is defined

$$\text{and } \boxed{\lim_{x \rightarrow a} f(x) = f(a)}$$

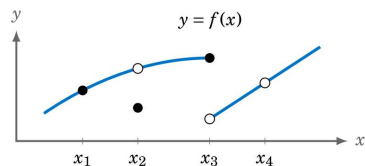


Figure 3.3.1 Continuous at x_1 , discontinuous at x_2, x_3 and x_4

Memorize

If f is a continuous function and $\lim_{x \rightarrow a} g(x)$ exists and is finite, then:

$$f\left(\lim_{x \rightarrow a} g(x)\right) = \lim_{x \rightarrow a} f(g(x)) \quad (3.5)$$

The same relation holds for one-sided limits.

Memorize the result

Every differentiable function is continuous.

Proof: If a function f is differentiable at $x = a$ then $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$ exists, so

$$\lim_{x \rightarrow a} (f(x) - f(a)) = \lim_{x \rightarrow a} (f(x) - f(a)) \cdot \frac{x - a}{x - a} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \cdot \lim_{x \rightarrow a} (x - a) = f'(a) \cdot 0 = 0$$

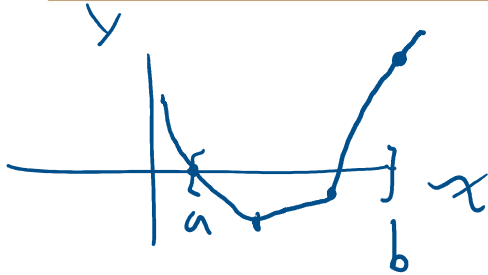
which means that $\lim_{x \rightarrow a} f(x) = f(a)$, i.e. f is continuous at $x = a$. ✓

$$\begin{aligned} h &= x - a \\ \Leftrightarrow x &= a + h \\ x \rightarrow a &\Leftrightarrow h \rightarrow 0 \end{aligned}$$

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h} = f'(a)$$

Memorize

Extreme Value Theorem: If f is a continuous function on a closed interval $[a, b]$ then f attains both a maximum value and a minimum value on that interval.



Copilot (not on a quiz or exam)

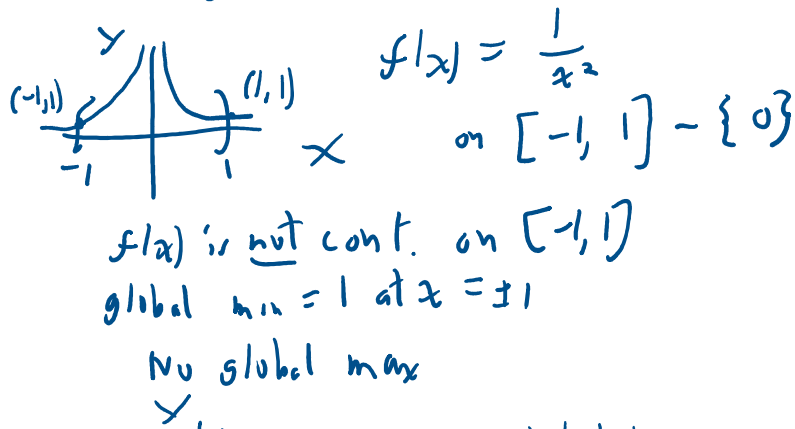
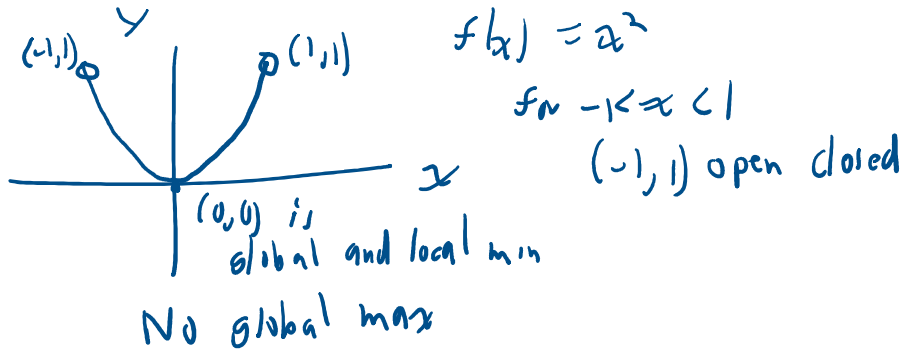
What is a tesseract. Does it have any practical value?

A tesseract is the four-dimensional analog of a cube—essentially, a 4D shape that extends the concept of a square (2D) and a cube (3D) into the fourth dimension. Mathematically, it's a type of hypercube with 8 cubic "faces" instead of the 6 square faces of a regular cube. Since we can't perceive four spatial dimensions directly, visual representations often show a tesseract as a projection or "shadow" in 3D space, much like how a cube casts a 2D shadow.

As for practical value, tesseracts don't have direct applications in our everyday 3D world, but they are incredibly useful in mathematics, physics, and computer science. For example:

- **Physics:** The concept of higher dimensions is essential in theories like string theory, which uses additional spatial dimensions to explain fundamental forces of the universe.
- **Data visualization:** In computer science, tesseracts can help represent and manipulate data in higher dimensions, such as in machine learning and data clustering.
- **Gaming and VR:** Tesseracts and higher-dimensional geometry inspire creative designs and mechanics in virtual worlds and games.
- **Literature and pop culture:** While not "practical" per se, tesseracts captivate the imagination, appearing in works like "A Wrinkle in Time" (as a way to traverse space-time) and various sci-fi stories.

Tesseracts might not revolutionize your daily routine, but they provide profound insights into abstract thinking and the nature of our universe. Do you find higher dimensions intriguing?

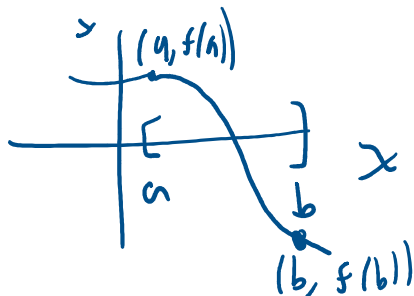




global max = 1
on $(-1, 1)$
global min = -1
on $(0, 1]$

f is not cont.
so Theorem does not apply,
but the conclusion of the Theorem is true

Intermediate Value Theorem: If f is a continuous function on a closed interval $[a, b]$ then f attains every value between $f(a)$ and $f(b)$.



Example 3.25

Show that there is a solution to the equation $\cos x = x$.

Let $f(x) = x - \cos x$
This is cont on $(-\infty, \infty)$

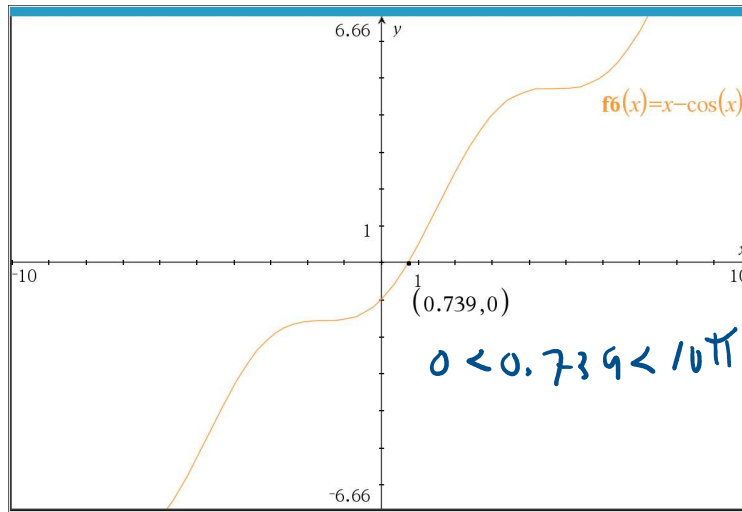
$$f(0) = 0 - \cos 0 = 0 - 1 = -1 < 0$$

$$f(10\pi) = 10\pi - \cos 10\pi$$

$$= 10\pi - 1$$

$$\approx 30 - 1 = 29 > 0$$

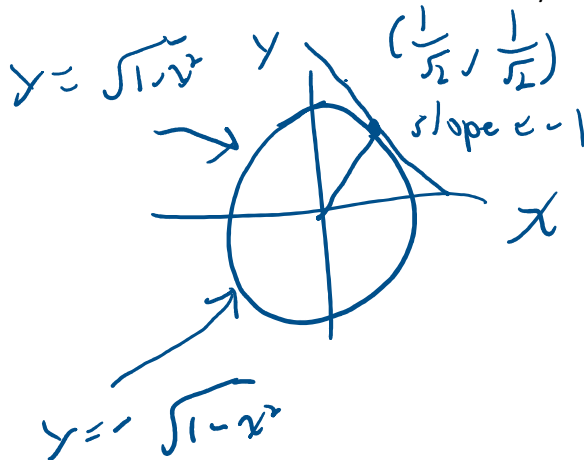
\therefore By IVT, $\exists c \in [0, 10\pi]$ with $f(c) = 0$



3.4

$$x^3 y^2 e^{\sin(xy)} = x^2 + xy + y^3, \text{ No solution found.}$$

SN could not solve this for y.



$$x^2 + y^2 = 1$$

$$y^2 = 1 - x^2$$

$$y = \pm \sqrt{1 - x^2}$$

$$\text{Find } \frac{dy}{dx} \text{ at } (x, y) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$y = (1 - x^2)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \left(\frac{1}{2}\right)(1 - x^2)^{-\frac{1}{2}}(-2x)$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}}$$

$$\frac{dy}{dx} \left(\frac{1}{\sqrt{2}}\right) = \frac{-\frac{1}{\sqrt{2}}}{\sqrt{1-\frac{1}{2}}} = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \boxed{-1}$$

or x

$$\frac{1}{\sqrt{1-\frac{1}{2}}} = \frac{\sqrt{2}}{\sqrt{\frac{1}{2}}} = \boxed{2}$$

implicit diff

$$x^2 + y^2 = 1$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(1)$$

$$2x + 2y \frac{dy}{dx} = 0$$

Solve algebraically for $\frac{dy}{dx}$

$$x + y \frac{dy}{dx} = 0$$

$$y \frac{dy}{dx} = -x$$

$$\boxed{\frac{dy}{dx} = -\frac{x}{y}}$$

$$\frac{dy}{dx}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \boxed{-1}$$