02-25-27 MTH 263

2.3 The Exponential and Natural Logarithm Functions

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2.4 General Exponential and Logarithmic Functions

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Memorize

$$\frac{d}{dx}\left(a^x\right) = (\ln a)\, a^x$$

$$\frac{d}{dx}\left(a^u\right) = (\ln a)\, a^u \cdot \frac{du}{dx}$$

Interesting:

$$a^x = e^{x \ln a} = e^{\ln a^x} = a^x$$

Memorize

$\log_a (bc) = \log_a b + \log_a c$	$a^b \cdot a^c = a^{b+c}$
$\log_a \left(\frac{b}{c}\right) = \log_a b - \log_a c$	$\frac{a^b}{a^c} = a^{b-c}$
$\log_a b^c = c \log_a b$	$\left(a^b\right)^c = a^{bc}$
$\log_a 1 = 0$	$a^0 = 1$

Memorize

$$\log_a x = \frac{\ln x}{\ln a}$$

$\ln a$

Find $\log_2 x$ in terms of $\ln x$

Let $y = \log_2 x$

transform log equation into an exp equation

$$2^y = x$$

$$\ln(2^y) = \ln(x)$$

$$y \ln(2) = \ln(x)$$

$$y = \frac{\ln(x)}{\ln(2)}$$

$$\therefore \boxed{\log_2(x) = \frac{\ln(x)}{\ln(2)}}$$ change of base formula

$$= \frac{\log(x)}{\log(2)}$$

$$\log_a x = \frac{\ln x}{\ln a}$$

$$\frac{d}{dx}(\log_a x) = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right)$$

$$= \frac{1}{\ln a}\frac{d}{dx}(\ln x)$$

$$= \frac{1}{\ln a}\left(\frac{1}{x}\right)$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\boxed{\frac{d}{dx}\left(\log_a x\right) = \frac{1}{x \ln a}}$$

Guichard

EXAMPLE 4.7.1 Compute the derivative of $f(x) = 2^x$.

$$2^x = e^{\ln(2^x)}$$

$$\frac{d}{dx}(2^x) = \frac{d}{dx}\left(e^{\ln(2^x)}\right)$$

Let $u = \ln(2^x)$

$$\frac{du}{dx} = \frac{1}{2^x}\frac{d}{dx}(2^x)$$

$$= \frac{d}{dx}\left(e^u\right)$$

$$= e^u \frac{du}{dx}$$

$$\frac{d}{dx}(2^x) = e^{\ln(2^x)}\left(\frac{1}{2^x}\right)\left(\frac{d}{dx}2^x\right)$$

$$= \frac{2^x}{2^x}\frac{d}{dx}(2^x)$$

$$= \frac{d}{dx}(2^x)$$

correct, but useless

$$2^x = e^{\ln(2^x)}$$

$$2^x = e^{x \ln 2}$$

$$a^{mn} = (a^m)^n$$

$$\frac{d}{dx}(2^x) = \frac{d}{dx}\left(e^{x \ln 2}\right)$$

$$= e^{x \ln 2}\frac{d}{dx}(x \ln 2)$$

$$= e^{x\ln 2} \frac{d}{dx}(x \ln 2)$$

$$= (e^{x\ln 2})(\ln 2)\frac{dx}{dx}$$

$$\frac{d}{dx}(2^x) = 2^x(\ln 2)$$

Guichard

EXAMPLE 4.7.3 Compute the derivative of $f(x) = x^x$

$$y = x^x$$

Find $\frac{dy}{dx}$ use log diff.

$$\ln(y) = \ln(x^x)$$

$$\ln(y) = x \ln|x|$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(x \ln|x|)$$

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}(\ln|x|) + \ln|x| \frac{d}{dx}(x)$$

$$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln|x|(1)$$

$$\frac{1}{y}\frac{dy}{dx} = 1 + \ln|x|$$

$$\frac{dy}{dx} = y(1 + \ln|x|)$$

$$\boxed{\frac{dy}{dx} = x^x(1 + \ln|x|)}$$

$d \quad d \qquad e^{x\ln(x)}$

$$\begin{aligned}\frac{d}{dx}x^x &= \frac{d}{dx}e^{x\ln x}\\ &= \left(\frac{d}{dx}x\ln x\right)e^{x\ln x}\\ &= (x\frac{1}{x}+\ln x)x^x\\ &= (1+\ln x)x^x\end{aligned}$$

$e^{x \ln(x)}$
$= e^{\ln(x^x)}$
$= x^x$

2.4

For Exercises 1-9, find the derivative of the given function.

2. $y = 2^{\ln 3x}$

$\frac{dy}{dx} = \frac{d}{dx}\left(2^{\ln(3x)}\right)$

Let $u = \ln(3x)$

$\frac{dy}{dx} = \frac{d}{dx}\left(2^u\right)$

$= 2^u (\ln 2) \frac{du}{dx}$

$= (\ln 2) 2^u \left(\frac{1}{3x}\right)\left(\frac{d}{dx}(3x)\right)$

$= (\ln 2) 2^u \left(\frac{1}{3x}\right)(3)$

$$\boxed{\frac{dy}{dx} = \left(\frac{\ln 2}{x}\right)\left(2^{\ln 3x}\right)}$$

$\frac{d}{} 2^{\ln(3x)} = \frac{2^{\ln 3+\ln x}}{} \ln 2$

$$\frac{d}{dx}2^{\ln(3x)} = \frac{2^{\ln 3+\ln x}}{x}\ln 2$$

2.4

8. $y = \log_2 4^{2x}$

$\log_b(a^r)$
$= r\log_b a$

$y = 2x\log_2 4$
$y = 2x\log_2(2^2)$
$y = 2x(2)$
$y = 4x$
$\frac{dy}{dx} = \frac{d}{dx}(4x) = \boxed{4}$

$y = \log_2 4^{2x}$

$$\boxed{\frac{d}{dx}(\log_a x) = \frac{1}{x\ln a}}$$

$$\frac{d}{dx}(\log_a u) = \frac{1}{u\ln a}\cdot\frac{du}{dx} = \frac{u'}{u\ln a}$$

$a = 2 \qquad u = 4^{2x}$

$\frac{d}{dx}(\log_2 4^{2x}) = \frac{1}{4^{2x}\ln 2}\cdot\frac{d}{dx}(4^{2x})$

$= \left(\frac{1}{4^{2x}\ln 2}\right)(4^{2x})(\ln 4)(2)$

$= \left(\frac{1}{\cancel{4^{2x}}\ln 2}\right)(\cancel{4^{2x}})(\ln 2^2)(2)$

$$\frac{d}{dx}\left(a^{u}\right) = (\ln a)\, a^{u} \cdot \frac{du}{dx}$$

$= (\cancel{4^{3x}} \ln 2)(\cancel{4} \;)(\ln 2)(2)$

$= \dfrac{2(2 \cancel{\ln 2})}{\cancel{\ln 2}}$

$= \boxed{4}$