12-08-25 MTH 263

MTH 263 student,
You are receiving this email because you are enrolled in a Fall 2025 section of MTH 263 at NOVA.
Every semester, the mathematics discipline assesses student learning in at least one class, for a particular topic. This semester, we have chosen MTH 263.
We assess what you have learned in a short quiz through google.
PLEASE RESPOND TO THIS SHORT SURVEY NO LATER THAN Tuesday, December 16.
You can access the survey by clicking on the link below.
https://docs.google.com/forms/d/e/1FAIpQLScKmvX85cl-vRWaNL1uI2CZ-pzrGA4aQhew31w8jqWeqelidg/viewform?usp=header
You may find it helpful to have pencil and paper handy. Calculators are permitted, but not necessary. You are expected to work independently without the use of any resources. there is no time limit on this quiz, but you should be able to finish in 5 – 10 minutes.
Before taking this short assessment, please ensure you know the following:
- your EMPLID (student ID number)
- the section of the MTH 263 course you are taking (i.e. 001A)
Data is collected to determine how we can improve instruction for particular topics. Your EMPLID is collected so we can disaggregate the data by various characteristics, such as your major. The answers and scores for individual students are not shared with your instructor.
A COPY OF YOUR RESPONSES WILL BE EMAILED TO YOUR NOVA EMAIL.

Thank you for your participation. Your responses will help us improve teaching and learning in MTH 263.
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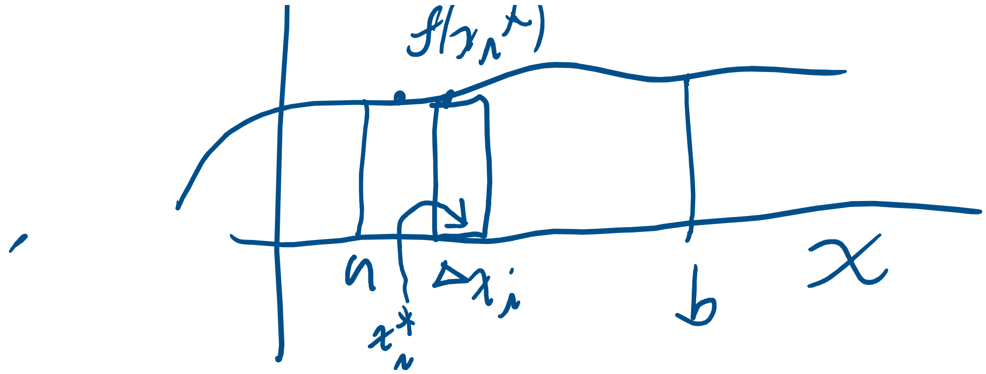
General review
Last class before final exam

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{\infty} f(x_i^*)\Delta x_i$$



7.5

For Exercises 3-15 prove the given identity. Your proofs can use other identities.

5. $\sinh A \cosh B = \frac{1}{2}(\sinh(A+B) + \sinh(A-B))$

$\sinh(u \pm v) = \sinh u \cosh v \pm \cosh u \sinh v$

$\frac{1}{2}(\sinh A \cosh B + \cosh A \sinh B)$

$+ \frac{1}{2}(\sinh A \cosh B - \cosh A \sinh B)$

$\sinh A \cosh B = \sinh A \cosh B$

Prove

$$\frac{d}{dx}(\sinh x) = \cosh x$$

Given

$$\sinh x = \frac{e^x - e^{-x}}{2} \quad \text{for all } x \qquad \cosh x = \frac{e^x + e^{-x}}{2} \quad \text{for all } x$$

$$\frac{d}{dx}(\sinh x) = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right)$$

$$= \left(\frac{1}{2}\right) \frac{d}{dx}(e^x - e^{-x})$$

$$=\left(\frac{1}{2}\right)\frac{d}{dx}\left(e^{x}-e^{-x}\right)$$

$$=\frac{1}{2}\left(e^{x}+e^{-x}\right)=\cosh x$$

Can you give me an optimization problem for my calculus class. No hint or solution.

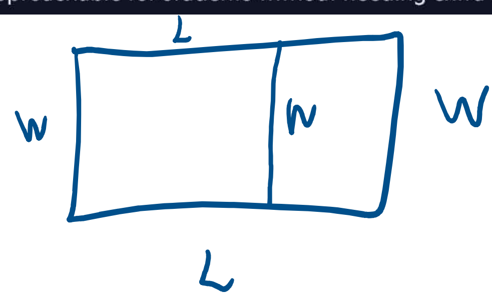
Here's a clean optimization problem you can present to your students:

📐 Optimization Problem

A farmer has 200 meters of fencing to enclose a rectangular field and divide it into two smaller rectangles by adding a fence parallel to one of the sides.

Question:
What dimensions of the overall rectangle will maximize the total area of the field?

This is a classic calculus optimization setup—simple to state, rich in application, and approachable for students without needing extra hints.



$$2L + 3W = 200\text{ m}$$

Now, please show the solution.

Perfect—let's solve the **farmer's fencing optimization problem** step by step:

📝 Restating the Problem

- Total fencing available: **200 m**
- Shape: A rectangle, divided into **two smaller rectangles** by one fence parallel to a side.
- Goal: Maximize the **area** of the overall rectangle.

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1. Define Variables

Let:

- L = length of the rectangle (the side parallel to the dividing fence)
- W = width of the rectangle

Because of the dividing fence, the fencing used is:

$$\text{Perimeter constraint: } 2W + 3L = 200$$

- Two widths (W)
- Three lengths (L) (two outer + one divider)

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2. Express Area

$$A = L \cdot W$$

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3. Solve Constraint for One Variable

From $2W + 3L = 200$:

$$W = \frac{200 - 3L}{2}$$

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4. Substitute into Area

$$A(L) = L \cdot \frac{200 - 3L}{2}$$

$$A(L) = \frac{200L - 3L^2}{2}$$

$$A(L) = 100L - 1.5L^2$$

5. Optimize

Take derivative:

$$A'(L) = 100 - 3L$$

defined for all L

Set $A'(L) = 0$:

$$100 - 3L = 0 \quad \Rightarrow \quad L = \frac{100}{3} \approx 33.3$$

6. Find Width

$$W = \frac{200 - 3(33.3)}{2} = \frac{200 - 100}{2} = 50$$

✅ Final Answer

- **Length:** L ≈ 33.3 m
- **Width:** $W = 50\,\text{m}$
- **Maximum Area:**

not required

$$A = L \cdot W = \frac{100}{3} \cdot 50 \approx 1666.7\,\text{m}^2$$

This is a nice example of how calculus turns a real-world constraint into a maximization problem.

Would you like me to also show a **graph of the area function** $A(L)$ so your students can visually see why the maximum occurs at $L = 100/3$?

How do we know that your critical value gives a maximum and not a minimum?

Excellent follow-up! In optimization problems, finding a critical point is only half the story—you need to check whether it's a maximum or minimum. Here's how we know in this case:

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1. Second Derivative Test

We had the area function:

$$A(L) = 100L - 1.5L^2$$

Take the second derivative:

$$A''(L) = -3$$

- Since $A''(L) = -3 < 0$, the function is **concave down everywhere**.
- That means any critical point is automatically a **maximum**, not a minimum.

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2. Graphical Intuition

- The area function is a **downward-opening parabola** in L.
- Downward parabolas peak at their vertex.
- So the critical value corresponds to the vertex, i.e. the maximum area.

✅ Conclusion

The critical value gives a maximum because the second derivative is negative, making the area function concave down.

Would you like me to **sketch the parabola of** $A(L)$ so your students can visually see why the maximum occurs at the vertex?

How do we know that the local max is actually a global max?

Great question—this is the kind of subtle distinction that really deepens students' understanding of optimization.

🧩 Why the local max is global here

1. **Domain restriction:**
   - The constraint $2W + 3L = 200$ forces both L and W to be nonnegative.
   - That means the feasible domain for L is .
2. **Nature of the function:**
   - The area function is

     $$A(L) = 100L - 1.5L^2$$

   - which is a **downward-opening parabola**.
   - A downward parabola has exactly one turning point, and that turning point is a maximum.

$W = 0 \Rightarrow L = \frac{200}{3}$

$0 \le L \le \frac{200}{3}$

compare $A(0)$, $A(50)$, $A(\frac{200}{3})$

3. **Boundary check:**
   - At $L = 0$, area $A = 0$.
   - At , area $A = 0$.
   - At the critical point , area is positive and largest.

At $L = \frac{200}{3}$

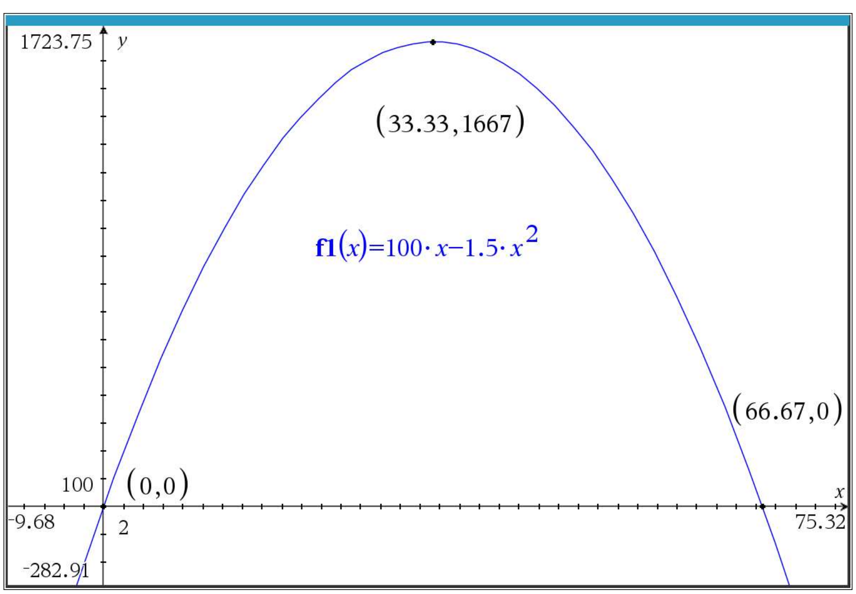
So the maximum at is not only local—it's the **global maximum** on the feasible domain.

✅ Teaching takeaway

You can emphasize to students:

- Always check the **endpoints** of the domain as well as the critical points.
- If the function is concave down (second derivative negative), the critical point is the unique global maximum.

On an exam, I would say "use calculus to maximize the area." The answer can be checked by considering the geometry of a parabola.

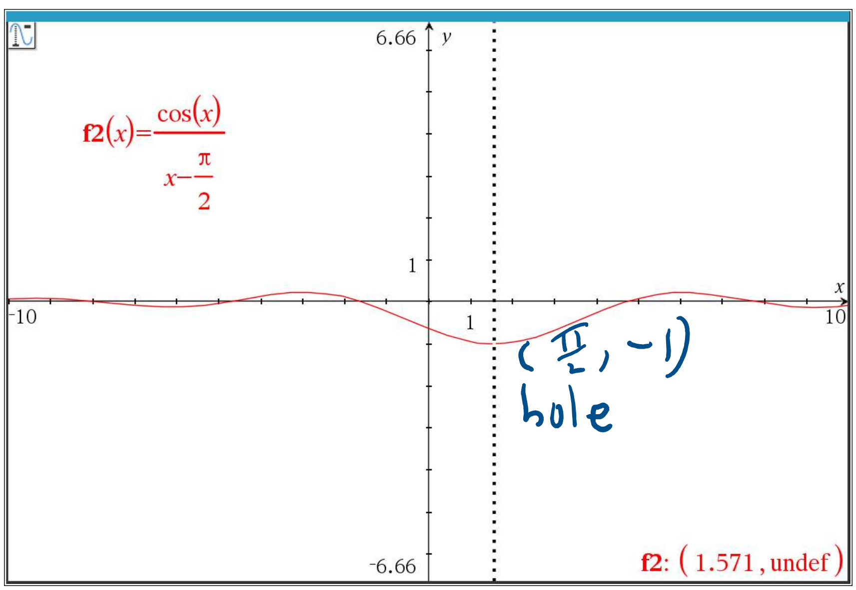


$$\lim_{x\to\frac{\pi}{2}} \frac{\cos x}{x-\frac{\pi}{2}} \overset{L}{=} \lim_{x\to\frac{\pi}{2}} \frac{\frac{d}{dx}(\cos x)}{\frac{d}{dx}\left(x-\frac{\pi}{2}\right)}$$

$$\frac{\cos\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}-\frac{\pi}{2}} = \left[\frac{0}{0}\right]$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{(-\sin x)}{1-0} = \lim_{x\to\frac{\pi}{2}} \frac{-\sin x}{1}$$

$$= -\sin\left(\frac{\pi}{2}\right) = \boxed{-1}$$



OK.We're still in calculus, could you please give me an example of L'Hopital's rule without giving me a hint or solution.

Sure! Try this one: Evaluate the limit as x approaches zero of (sin x) over x. That's a classic scenario where L'Hospital's Rule comes in handy.

$$\lim_{x \to 0} \frac{\sin(x)}{x} \left[\frac{0}{0}\right] \overset{(L)}{=} \lim_{x \to 0} \frac{\frac{d}{dx}(\sin(x))}{\frac{d}{dx}(x)}$$

$$= \lim_{x \to 0} \frac{\cos(x)}{1} = \lim_{x \to 0} \cos(x)$$

$$= \cos(0) = \boxed{1}$$