

7 Analytic Geometry and Plain Curves**7.5 Hyperbolic Functions**page 246: 1 (for $\sinh(-x)$), 12, 21, 27**8 Applications of Integrals****8.1 Area Between Curves**

page 266: 1, 5, 7, 9

8.2 Average Value of a Function

page 271: 1, 4, 7, 10

7.5: 27

27. The general solution of the differential equation $y'' = a^2 y$ is $y(t) = y_1(t) = c_1 e^{at} + c_2 e^{-at}$, where a is a positive constant, and c_1 and c_2 are arbitrary constants.

(a) Verify that $y(t) = y_2(t) = k_1 \cosh at + k_2 \sinh at$ is also a solution of $y'' = a^2 y$.

(b) Show that for any c_1 and c_2 , $y_1(t) = c_1 e^{at} + c_2 e^{-at}$ can be written as $y_1(t) = k_1 \cosh at + k_2 \sinh at$ for some constants k_1 and k_2 in terms of c_1 and c_2 .

Show that $y_1(t)$ is a solution of the differential equation.

$$\begin{aligned}
 y_1(t) &= c_1 e^{at} + c_2 e^{-at} \\
 y_1'(t) &= c_1 a e^{at} - c_2 a e^{-at} \\
 y_1''(t) &= c_1 a^2 e^{at} + c_2 a^2 e^{-at} \\
 a^2 y_1(t) &= c_1 a^2 e^{at} + c_2 a^2 e^{-at}
 \end{aligned}$$

$$\therefore y_1''(t) = a^2 y_1(t)$$

$\therefore y_1(t)$ is the general solution of the differential equation

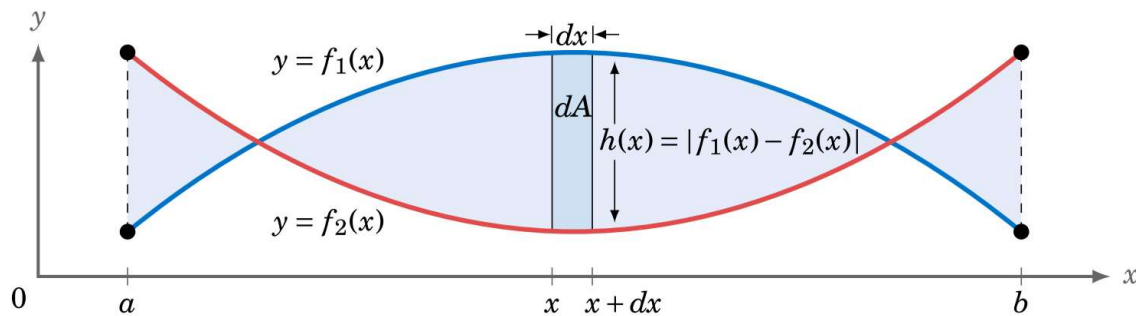
(a)
$$\begin{aligned}
 \frac{d}{dx}(\sinh x) &= \cosh x \\
 \frac{d}{dx}(\cosh x) &= \sinh x
 \end{aligned}$$

$$\text{let } y_2(t) = k_1 \cosh(at) + k_2 \sinh(at)$$

$$y_2'(t) = a k_1 \sinh(at) + a k_2 \cosh(at)$$

$$\begin{aligned}
 y_2'(t) &= a k_1 \sinh(at) + a k_2 \cosh(at) \\
 y_2''(t) &= a^2 k_1 \cosh(at) + a^2 k_2 \sinh(at) \\
 a^2 y_2(t) &= a^2 k_1 \cosh(at) + a^2 k_2 \sinh(at) \\
 \therefore y_2''(t) &= a^2 y_2(t) \\
 \therefore y_2(t) &\text{ is also a solution} \\
 &\text{ to the diff. eq.}
 \end{aligned}$$

8.1

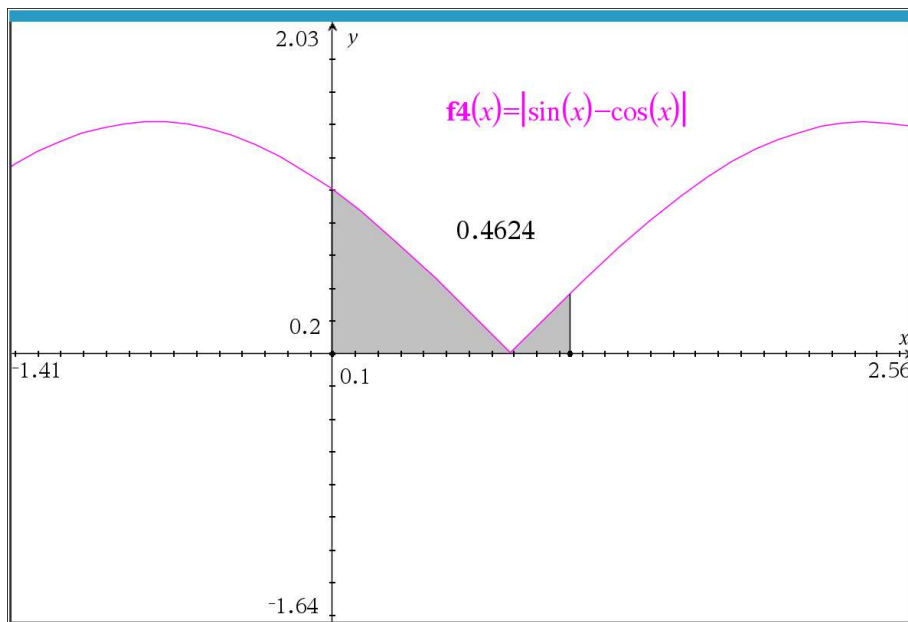


memorize

The area A between two curves $y = f_1(x)$ and $y = f_2(x)$ over an interval $[a, b]$ is:

$$A = \int_a^b dA = \int_a^b |f_1(x) - f_2(x)| dx \quad (8.1)$$

The interval $[a, b]$ can be replaced by any interval—finite or infinite—over which the integral is defined. Neither curve is required to be above the x -axis.



$\frac{4 \cdot \sqrt{2} - 3 - \sqrt{3}}{2}$	$\frac{-(\sqrt{3} - 4 \cdot \sqrt{2} + 3)}{2}$
$\frac{-(\sqrt{3} - 4 \cdot \sqrt{2} + 3)}{2} \rightarrow \text{Decimal}$	0.462402

Note: TI-84 gave 0.4634584

glitch

Supplied

The area A between two polar curves $r = r_1(\theta)$ and $r = r_2(\theta)$ for $\alpha \leq \theta \leq \beta$ is:

$$A = \int_{\alpha}^{\beta} \frac{1}{2} |r_1^2 - r_2^2| d\theta \quad (8.2)$$

Copilot

Give me an example, without working it out, of finding the area bounded by two curves.

Sure! Here's a classic example:

Find the area bounded by the curves

$y = x^2$ and $y = 2x + 3$

This setup involves a parabola and a straight line. To find the area between them, you'd typically:

- Determine the points of intersection.
- Set up an integral with the top curve minus the bottom curve.
- Integrate over the interval defined by the intersection points.

intersection points

Set up an integral with the top curve minus the bottom curve.

- Integrate over the interval defined by the intersection points.

intersection points

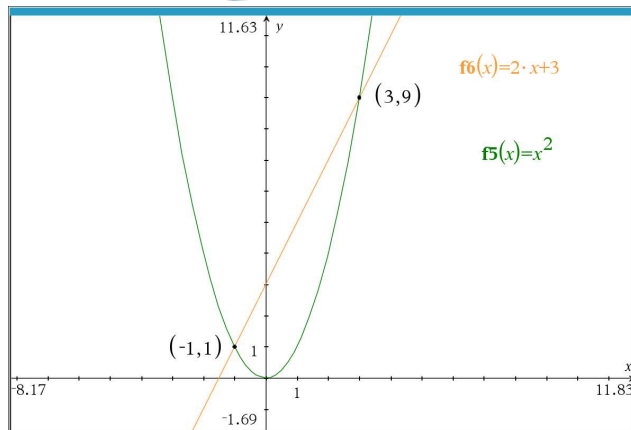
$$x^2 = 2x + 3$$

$$x^2 - 2x - 3 = 0$$

$$(x + 1)(x - 3) = 0$$

$$\boxed{x = -1, 3}$$

$$\Rightarrow \boxed{y = 1, 9}$$



Let A = area between curves

$$A = \int_{-1}^3 |2x + 3 - x^2| dx$$

$$= \int_{-1}^3 (2x + 3 - x^2) dx$$

$$= \left[x^2 + 3x - \frac{x^3}{3} \right]_{-1}^3$$

$$= (9 + 9 - 9) - \left(1 - 3 + \frac{1}{3} \right)$$

$$= 9 - \left(-2 + \frac{1}{3} \right)$$

$$= 11 - \frac{1}{3} = \boxed{\frac{32}{3}} \approx 10.67$$

$$32/3 = 10.6667$$

