

5.3 The Fundamental Theorem of Calculus
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5.4 Integration by Substitution
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5.3: 12

Exercises

A

For Exercises 1-12, evaluate the given definite integral.

12. $\int_{-2}^2 \frac{x^3 e^{x^2}}{\cos 2x} dx$

Let $f(x) = \frac{x^3 e^{x^2}}{\cos(2x)}$

$$f(-x) = \frac{(-x)^3 e^{(-x)^2}}{\cos(-2x)} = \frac{(\text{odd})(\text{even})}{\text{even}} = \text{odd}$$

$$= \frac{-x^3 e^{x^2}}{\cos(2x)}$$

$$= -f(x)$$

$\therefore f$ is odd

$$\therefore \int_{-2}^2 f(x) dx = \boxed{0}$$

5.3

13. Show that $\ln x = \int_1^x \frac{1}{t} dt$ for all $x > 0$.

Power Formula: $\int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} & \text{if } n \neq -1 \\ \ln|x| & \text{if } n = -1 \end{cases}$

$$\int x^{-1} dx = \int \left(\frac{1}{x}\right) dx = \ln|x|$$

$$\int_1^x \frac{1}{t} dt = \ln|t| \Big|_1^x$$

$$\begin{aligned}
 &= \ln|x| - \ln(1) \\
 &= \ln|x| - 0 \\
 &= \ln|x| \\
 &= \ln(x) \text{ for } x > 0
 \end{aligned}$$

5.4

Rule of thumb: let u = any complicated part of the integrand, something under a radical, or the denominator of a fractional expression.
Then, check if you see du anywhere.

supplied

For any constant a ,

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx. \quad (5.6)$$

$$\begin{aligned}
 &\text{Let } u = a - x \\
 &\Rightarrow du = -dx \Rightarrow dx = -du \\
 &x=0 \Rightarrow u = a \\
 &x=a \Rightarrow u = 0 \\
 &\therefore \int_0^a f(a-x) dx \\
 &= \int_a^0 f(u) (-du) \\
 &= - \int_a^0 f(u) du \\
 &= \int_0^a f(u) du \\
 &= \int_0^a f(x) dx
 \end{aligned}$$

Guichard

$$\int \underline{2x} \cos(\underline{x^2}) dx.$$

$$\int \underline{2x} \cos(x^2) \underline{dx}.$$

$$\text{Let } u = \cos(x^2)$$

$$\Rightarrow du = d(\cos(x^2)) \\ = \underbrace{(-\sin(x^2))}_{\text{not in integrand}} (2x) dx$$

$$\text{Let } u = x^2$$

$$\Rightarrow du = \underline{2x dx}$$

$$\text{Let } I = \int 2x \cos(x^2) dx$$

$$I = \int \cos(u) du$$

$$= \sin(u) + C$$

$$\boxed{I = \sin(x^2) + C}$$

$$\text{check } \frac{d}{dx} (\sin(x^2) + C)$$

$$= \frac{d}{dx} (\sin(x^2)) + \frac{d}{dx} (C)$$

$$= (\cos(x^2)) 2x + 0$$

$$= 2x \cos(x^2) \quad \checkmark$$

Guichard

Exercises 8.1.

Find the antiderivatives or evaluate the definite integral in each problem.

$$4. \int \frac{1}{\sqrt[3]{1-5t}} dt = I$$

$$\text{Let } u = 5t$$

$$\Rightarrow du = 5 dt \Rightarrow dt = \frac{du}{5}$$

$$I = \frac{1}{5} \int \frac{1}{\sqrt[3]{1-u}} du$$

$$\text{Let } w = 1-u$$

$$\Rightarrow dw = -du$$

$$\Rightarrow du = -dw$$

$$I = -\frac{1}{5} \int \frac{1}{\sqrt[3]{w}} dw$$

$$= -\frac{1}{5} \int w^{-\frac{1}{3}} dw$$

$$= -\frac{1}{5} \left[\frac{3w^{\frac{2}{3}}}{\frac{2}{3}} \right] + C$$

$$= -\frac{3}{10} \left[(1-u)^{\frac{2}{3}} \right] + C$$

$$\boxed{I = -\frac{3}{10} (1-5t)^{\frac{2}{3}} + C}$$

$$\begin{aligned}
 & \frac{d}{dt} \left(-\frac{3}{10} (1-st)^{2/3} + C \right) \\
 &= \cancel{1} \left(-\frac{3}{10} \right) \left(\frac{2}{3} \right) (1-st)^{-1/3} \cancel{(-s)} \\
 &= \boxed{\frac{1}{3\sqrt[3]{1-st}}}
 \end{aligned}$$

$$I = \int \frac{1}{3\sqrt[3]{1-st}} dt$$

$$u = \sqrt[3]{1-st} = (1-st)^{1/3}$$

$$du = \left(\frac{1}{3} \right) (1-st)^{-2/3} (-s) dt$$

$$du = \left(-\frac{s}{3} \right) (1-st)^{-2/3} dt$$

$$\Rightarrow dt = \left(-\frac{3}{s} \right) (1-st)^{2/3} du$$

$$I = \int \frac{1}{3\sqrt[3]{1-st}} \left(-\frac{3}{s} \right) (1-st)^{2/3} du$$

$$\begin{aligned}
 I &= \left(-\frac{3}{5}\right) \int (1-st)^{\frac{2}{3}-\frac{1}{3}} du \\
 &= \left(-\frac{3}{5}\right) \int (1-st)^{\frac{1}{3}} du \\
 &= -\frac{3}{5} \int u du \\
 &= -\frac{3}{5} \left(\frac{u^2}{2} \right) + C \\
 &= -\frac{3}{10} u^2 + C \\
 &= -\frac{3}{10} \left(\sqrt[3]{1-st} \right)^2 + C
 \end{aligned}$$

Guichard 8.1

$$15. \int_3^4 \frac{1}{(3x-7)^2} dx = I$$

$$\begin{aligned}
 \text{Let } u &= 3x-7 \\
 \Rightarrow du &= 3 dx \Rightarrow dx = \frac{du}{3}
 \end{aligned}$$

$$x = 3 \Rightarrow u = 3(3) - 7 = 9 - 7 = 2$$

$$x = 4 \Rightarrow u = 3(4) - 7 = 12 - 7 = 5$$

$$.15 \quad .15$$

$$I = \frac{1}{3} \int_2^5 \frac{1}{u^2} du$$

$$= \frac{1}{3} \int_2^5 u^{-2} du$$

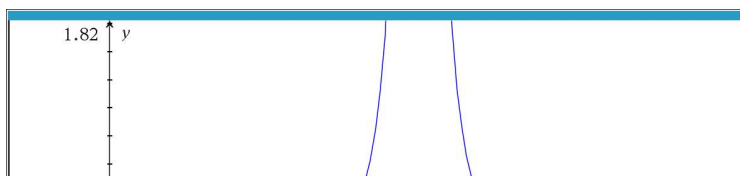
$$= \frac{1}{3} \left[\frac{u^{-1}}{-1} \right]_2^5$$

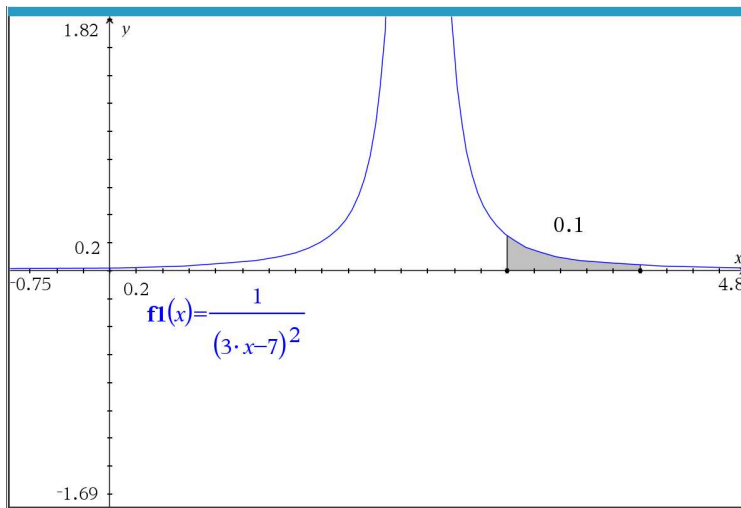
$$= -\frac{1}{3} \left[\frac{1}{5} - \frac{1}{2} \right]$$

$$= -\frac{1}{3} \left[\frac{2}{10} - \frac{5}{10} \right]$$

$$= -\frac{1}{3} \left(-\frac{3}{10} \right)$$

$$\boxed{I = \frac{1}{10}}$$





Alternative presentation

$$\int_3^4 \frac{1}{(3x-7)^2} dx = I$$

$$\text{Let } u = 3x - 7 \\ \Rightarrow du = 3 dx \Rightarrow dx = \frac{du}{3}$$

$$I = \frac{1}{3} \int_{x=3}^4 u^{-2} du$$

$$= \frac{1}{3} \left[\frac{u^{-1}}{-1} \right]_{x=3}^4$$

$$= -\frac{1}{3} \left[\frac{1}{u} \right]_{x=3}^4$$

$$\begin{aligned}
 & \Rightarrow \quad x=3 \\
 & = -\frac{1}{3} \left[\frac{1}{3x-7} \right]_3^4 \\
 & = -\frac{1}{3} \left[\frac{1}{3(4)-7} - \frac{1}{(3)(3)-7} \right] \\
 & = -\frac{1}{3} \left[\frac{1}{5} - \frac{1}{2} \right] \\
 & = -\frac{1}{3} \left[\frac{2}{10} - \frac{5}{10} \right] \\
 & = -\frac{1}{3} \left[-\frac{3}{10} \right] = \boxed{\frac{1}{10}}
 \end{aligned}$$

Bonus quiz 3

1. Evaluate $\int (3x - 2x^2 + 5)dx$.

$$\boxed{\frac{3x^2}{2} - \frac{2x^3}{3} + 5x + C}$$

2. Evaluate $\int_{-3}^3 \frac{\sin(x)}{e^{x^2}} dx$ analytically.

$$\text{Let } f(x) = \frac{\sin(x)}{e^{x^2}}$$

$$f(-x) = \frac{\sin(-x)}{e^{(-x)^2}} = -\frac{\sin(x)}{e^{x^2}} = -f(x)$$

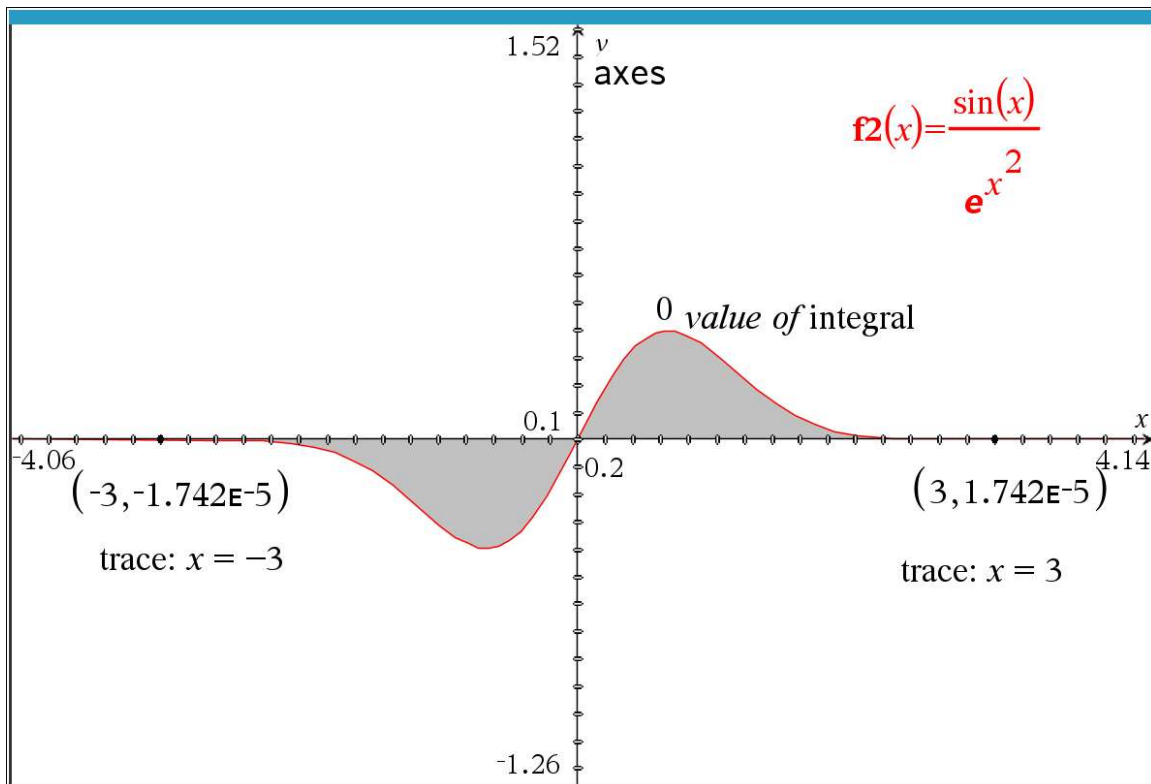
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$$f(-x) = \frac{\sin(-x)}{e^{(-x)^2}} = \frac{-\sin x}{e^{x^2}} = -f(x)$$

$\therefore f$ is odd

$$\int_{-3}^3 f(x) dx = 0$$

3. Evaluate #2 graphically. Sketch a labeled graph.



Evaluate $\int_1^2 (x+1)^{99} dx = I$

Let $u = x+1$

$\Rightarrow du = dx$

$x=1 \Rightarrow u = 1+1 = 2$

$x=2 \Rightarrow u = 2+1 = 3$

$$I = \int_2^3 u^{99} du = \left[\frac{u^{100}}{100} \right]_2^3$$

$$I = \int_2^3 u^{99} du = \left. \frac{u^{100}}{100} \right|_2^3$$

$$= \frac{3^{100} - 2^{100}}{100} \approx 5.15 \times 10^{45}$$

$\frac{3^{100} - 2^{100}}{100}$	$\frac{20615100829280453190752421181495674848216172665}{4}$
20615100829280453190752421181495674848216172665	5.15378E45
Decimal	

$\int_1^2 (x+1)^{99} dx$	$\frac{20615100829280453190752421181495674848216172665}{4}$
20615100829280453190752421181495674848216172665	5.15378E45
Decimal	