11-10-25 MTH 263

5.2 The Definite Integral
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5.3 The Fundamental Theorem of Calculus
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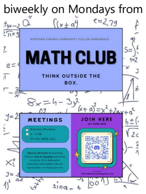
Math Club: Our club is open to all students, regardless of their current math level. We provide a fun and engaging space to explore math topics, connect with peers, and learn about new opportunities. We meet biweekly on Mondays from 3:00 PM to 4:00 PM in CT-121 (MDE Lab).

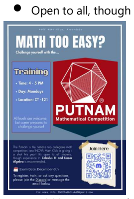


2nd Annual Integration Bee: This is a fun and challenging competition with over **$300** in prizes. It's a great event for students to test their calculus skills and even challenge their professors! We'll have snacks, and all are welcome to attend, even just to watch.

- Date: Friday, November 21st
- Time: 3:00 PM
- Location: CA - 302 (Annandale Campus)
- Calculus II experience is recommended for competitors.

Putnam Mathematical Competition: We are actively recruiting and training students for the Putnam, the nation's top collegiate math competition. This is a fantastic opportunity for students seeking a serious challenge.

- Exam Date: Saturday, December 6th
- Training Sessions: Mondays from 4:00 PM to 5:00 PM in CT-121
- Open to all, though experience in Calculus III and Linear Algebra is recommended.



We would be very grateful if you could forward this email, along with the attached flyers, to the Annandale math faculty. Their support is invaluable for getting students involved.

Thank you for your time and for supporting student engagement in mathematics.

Best regards,

Emiliano Mercado
NVCCMathClubAN@gmail.com

5.3
Memorize

Fundamental Theorem of Calculus: Suppose that a function f is differentiable on $[a, b]$. Then:

(I) The function $A(x)$ defined on $[a, b]$ by

$$A(x) = \int_a^x f(t)\,dt$$

is differentiable on $[a, b]$, and

$$A'(x) = f(x)$$

for all x in $[a, b]$.

(II) If F is an antiderivative of f on $[a, b]$, i.e. $F'(x) = f(x)$ for all x in $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)\,.$$

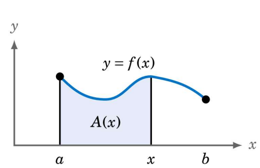


Figure 5.3.1 The area function $A(x) = \int_a^x f(t)\,dt$

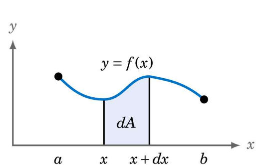


Figure 5.3.2 $dA = A(x + dx) - A(x)$

From proof of part II.

To prove Part II of the theorem, let $F(x)$ be an antiderivative of $f(x)$ over $[a,b]$. Since $A(x) = \int_a^x f(\overset{t}{\cancel{x}})\,d\overset{t}{\cancel{x}}$ is also an antiderivative of $f(x)$ over $[a,b]$ by Part I of the theorem, then $A(x)$ and $F(x)$ differ by a constant C over $[a,b]$. In other words:

$\int_a^x f(t)\,dt$, t = dummy variable

$t \neq x$

Notation

$$F(x)\Big|_a^b = F(b) - F(a)$$

$$F(x)\Big]_a^b = F(b) - F(a)$$

Memorize

If f is an odd function, i.e. $f(-x) = -f(x)$ for all x, then

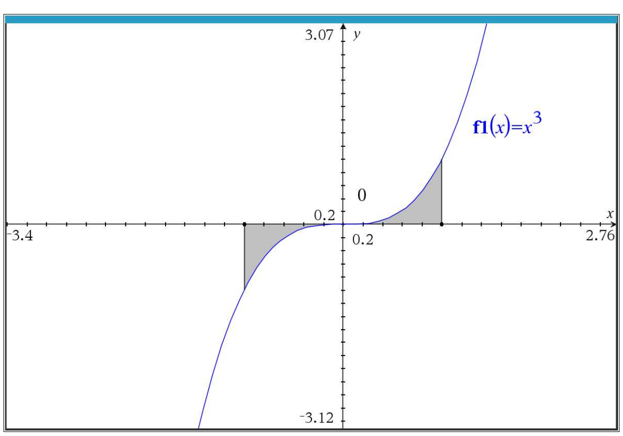
$$\int_{-a}^{a} f(x)\,dx = 0$$

for all $a > 0$ such that f is continuous on $[-a,a]$.

$$\int_{-a}^{a} x^3\,dx = \frac{x^4}{4}\Big]_{-a}^{a}$$

$$= \frac{a^4}{4} - \frac{(-a)^4}{4}$$

$$= \frac{a^4}{4} - \frac{a^4}{4} = 0$$



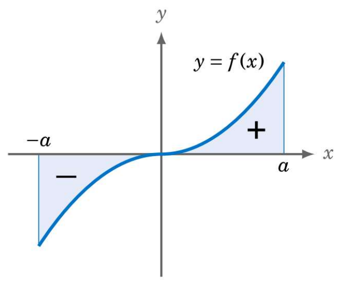


Figure 5.3.4 Odd function f over $[-a,a]$

If f is an even function, i.e. $f(-x) = f(x)$ for all x, then

$$\int_{-a}^{a} f(x)\,dx \;=\; 2\int_{0}^{a} f(x)\,dx$$

for all $a > 0$ such that f is continuous on $[-a,a]$.

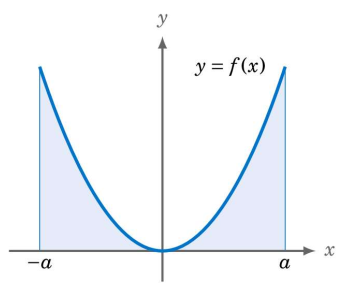


Figure 5.3.5 Even function f over $[-a,a]$

$$\int_{-a}^{a} x^2\,dx = \left.\frac{x^3}{3}\right]_{-a}^{a}$$

$$= \frac{a^3}{3} - \frac{(-a)^3}{3}$$

$$= \frac{a^3}{3} - \frac{(-a^3)}{3}$$

$$= \frac{a^3}{3} + \frac{a^3}{3}$$

$$= 2\left(\frac{a^3}{3}\right)$$

Memorize

Let f and g be continuous functions on $[a,b]$ and let k be a constant. Then:

1. $\displaystyle\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$

2. $\displaystyle\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

3. $\displaystyle\int_a^b (f(x)-g(x))\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

Memorize

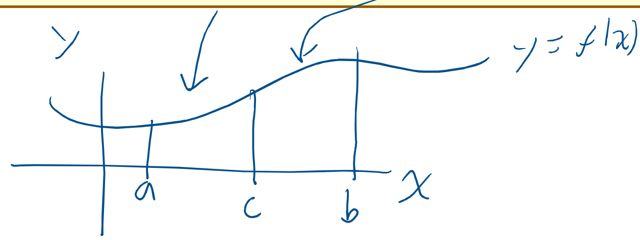
Let f be a continuous function on $[a,b]$ and suppose that $a<c<b$. Then:

(1) $\displaystyle\int_a^a f(x)\,dx = 0$

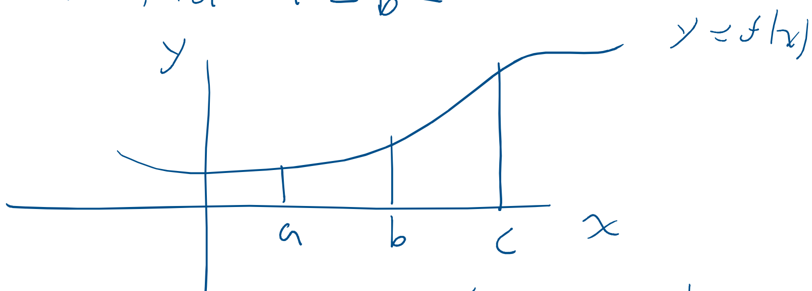
(2) $\displaystyle\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

(3) $\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

Note we don't need $a<c<b$



Now, let $a<b<c$



Show $\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

$$\underbrace{\int_a^b f(x)\,dx + \int_b^c f(x)\,dx}_{} - \int_b^c f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^b f(x)\,dx$$

Memorize

Chain Rule for integrals: Let f be a continuous function on an interval I containing $x = a$, and let $g(x)$ be a differentiable function on I. If

$$F(x) = \int_a^{g(x)} f(t)\, dt \quad \text{for all } x \text{ in } I$$

then $F'(x) = f(g(x)) \cdot g'(x)$ for all x in I.

Let $F(x) = \int_2^{x^2} t\, dt$

$$F(x) = \left.\frac{t^2}{2}\right]_2^{x^2} = \frac{x^4}{2} - \frac{4}{2}$$

$$= \boxed{\frac{x^4}{2} - 2}$$

$$F'(x) = \frac{d}{dx}\left(\frac{x^4}{2} - 2\right) = \left(\frac{1}{2}\right)(4x^3) - 0$$

$$= \boxed{2x^3}$$

$$F'(x) \stackrel{?}{=} f(g(x)) \cdot g'(x)$$

$$f(x) = x$$

$$f(g(x)) = g(x) = x^2$$

$$g'(x) = 2x$$

$$f(g(x)) \cdot g'(x) = x^2(2x)$$

$$= \boxed{2x^3}$$

Let $F(x) = \int_0^x \ln\left(\frac{\sqrt{\sin t}}{e^t}\right) dt$

$0 < x < \frac{\pi}{2}$

Find $F'(x) = \ln\left(\frac{\sqrt{\sin x}}{e^x}\right)$

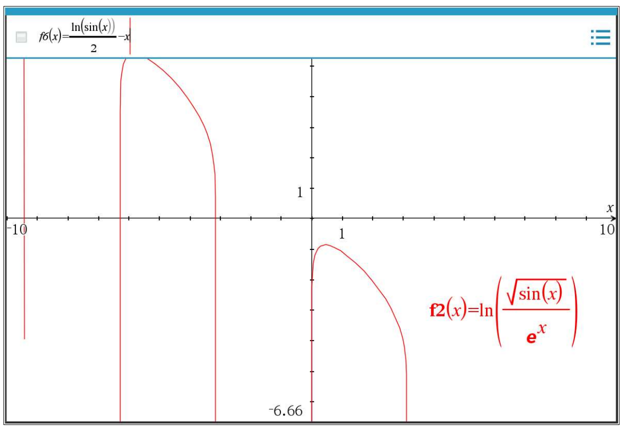
$$\frac{d}{dx}\left(\int_0^x \ln\left(\frac{\sqrt{\sin(t)}}{\mathbf{e}^t}\right)dt\right) \qquad \frac{\ln(\sin(x))}{2}-x$$

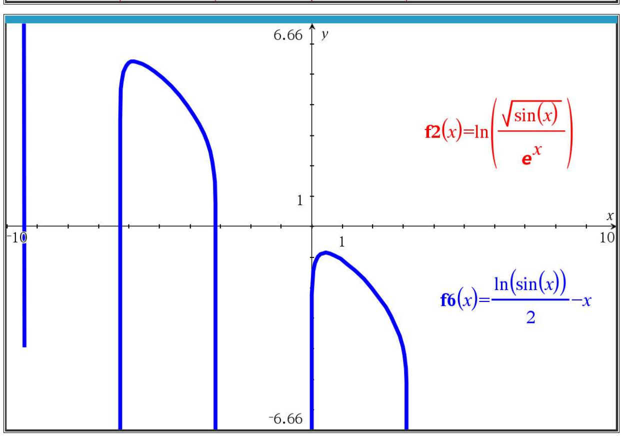
Warning

Domain of the result might be larger than the domain of the input.

OK

$$\ln\left(\frac{\sqrt{\sin x}}{e^x}\right) = \ln(\sqrt{\sin x}) - \ln(e^x)$$
$$= \ln\left((\sin x)^{\frac{1}{2}}\right) - x$$
$$= \left(\frac{1}{2}\right) \cdot \ln(\sin x) - x$$





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Compute the values of the integrals:

11. $\displaystyle\int_1^4 (t^2 + 3t)\,dt$ 　　 fn Int$(x^2+3x, x, 1, 4)$

$$= \left.\frac{t^3}{3} + \frac{3t^2}{2}\right]_1^4 = \frac{4^3}{3} + \frac{3(4^2)}{2}$$

$$= \frac{t^3}{3} + \frac{3t^2}{2}\Big]_1 = \frac{4}{3} + \frac{3(4)}{2} - \left[\frac{1}{3} + \frac{3}{2}\right]$$

$$= \frac{64}{3} + 24 - \frac{1}{3} - \frac{3}{2}$$

$$= \frac{63}{3} + \frac{24(6)}{6} - \frac{9}{6}$$

64/3+24-1/3-3/2=43.5

17. Find the derivative of $G(x) = \int_1^x t^2 - 3t\,dt \Rightarrow$

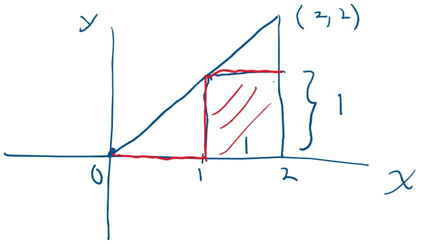
$$G'(x) = x^2 - 3x$$

$$\int_{-\pi}^{\pi} x^5\,dx = 0$$

Your Name MTH 263 quiz 8 $\int_a^b f(x)\,dx \approx \sum_i f(x_i)\Delta x$

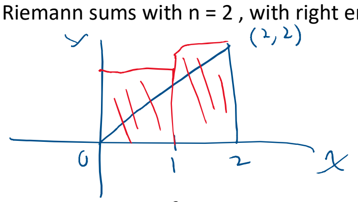
Make sketches for #1 and #2.

1. Use Riemann sums with n = 2 , with left endpoints, to estimate $\int_0^2 x\,dx$.



$$\int_0^2 x\,dx \approx (f(0))(1) + (f(1))(1) = (0)(1) + (1)(1) = \boxed{1}$$

2. Use Riemann sums with n = 2 , with right endpoints, to estimate $\int_0^2 x\,dx$.



$$\int_0^2 x\,dx \approx (f(1))(1) + (f(2))(1) = (1)(1) + (2)(1) = \boxed{3}$$

3. Find exact value of $\int_0^2 x\,dx$ using the fundamental theorem of calculus.

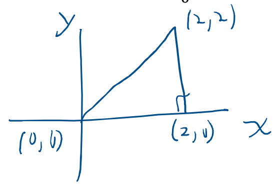
$$\int_0^2 x\,dx = \frac{x^2}{2}\Big]_0^2 = \frac{2^2}{2} - \frac{0}{2} = \frac{4}{2} = \boxed{2}$$

4. Compare the above three results.

$1 < 2 < 3$

under estimate — exact value — over estimate

5. Find exact value of $\int_0^2 x dx$ using elementary geometry.



$$\int_0^2 x\,dx = \text{area of triangle}$$
$$= \left(\tfrac{1}{2}\right)(\text{base})(\text{height})$$
$$= \left(\tfrac{1}{2}\right)(2)(2) = \boxed{2}$$