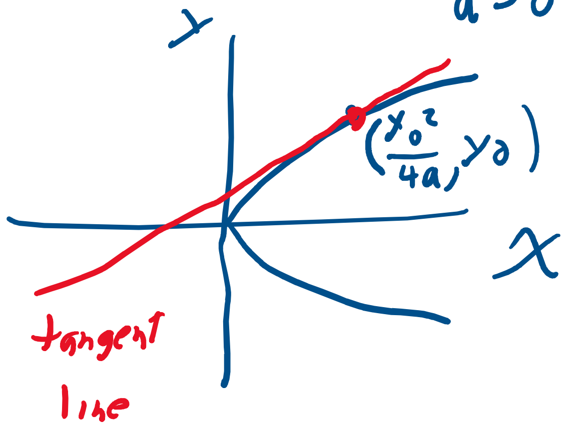
3.3 Continuity
page 88: 1, 5, 15, 25, 29
3.4 Implicit Differentiation
page 91: 1, 7, 13, 14

3.5 Related Rates
page 93: 1, 3, 6, 10

3.4:14

14. Show that at every point (x_0, y_0) on the curve $y^2 = 4ax$, the equation of the tangent line to the curve is $yy_0 = 2a(x + x_0)$.

$a > 0$



$y^2 = 4ax$
Let $y = y_0$
$\Rightarrow y_0^2 = 4ax_0$
$\Rightarrow x_0 = \frac{y_0^2}{4a} \Rightarrow y_0^2 = 4ax_0$

$y^2 = 4ax$
differentiate implicitly
to find $\frac{dy}{dx}$

$y^2 = 4ax \Rightarrow y = \pm\sqrt{4ax}$

$y = \pm 2\sqrt{ax}$

$2y\frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{4a}{2y}$

$\frac{dy}{dx} = \frac{2a}{y}$

$\frac{dy}{dx}\left(\frac{y_0^2}{}, \right) = \frac{2a}{}$

$$\boxed{\frac{dy}{dx}\left(\frac{y_0^2}{4a}, y_0\right) = \frac{2a}{y_0}}$$

point-slope equation of tangent line

$$y - y_0 = m(x - x_0)$$

$$y - y_0 = \frac{2a}{y_0}\left(x - \frac{y_0^2}{4a}\right)$$

convert this into $yy_0 = 2a(x+x_0)$

$$yy_0 - y_0^2 = 2a\left(x - \frac{y_0^2}{4a}\right)$$

$$yy_0 = 2a\left(x - \frac{y_0^2}{4a}\right) + y_0^2$$

$$yy_0 = 2a(x - x_0) + y_0^2$$

$$yy_0 = 2a(x - x_0) + 4a x_0$$

$$yy_0 = 2ax - 2ax_0 + 4a x_0$$

$$yy_0 = 2ax + 2a x_0$$

$$\boxed{yy_0 = 2a(x + x_0)}$$

3.4: 7

For Exercises 1-9, use implicit differentiation to find $\frac{dy}{dx}$.

7. $\cos(xy) = \sin(x^2y^2)$

$$\frac{d}{dx}\left(\cos(xy)\right) = \frac{d}{dx}\left(\sin(x^2y^2)\right)$$

$$\left(-\sin(xy)\right)\frac{d}{dx}(xy) = \left(\cos(x^2y^2)\right)\frac{d}{dx}(x^2y^2)$$

$$\left(-\sin(xy)\right)\left(x\frac{dy}{dx} + y\right) = \left(\cos(x^2y^2)\right)\left(x^2(2y)\frac{dy}{dx} + 2xy^2\right)$$

solve algebraically for $\frac{dy}{dx}$

finish at home

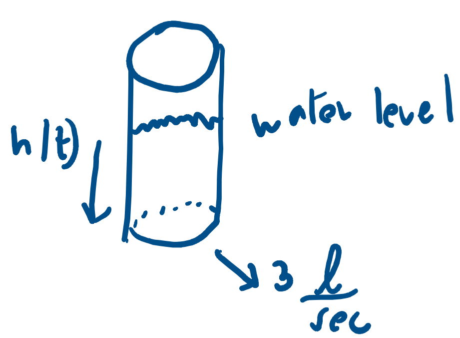
3.5 related rates

Guichard

Exercises 6.2.

2. A cylindrical tank standing upright (with one circular base on the ground) has radius 1 meter. How fast does the water level in the tank drop when the water is being drained at 3 liters per second? ⇒

→ $r = 1$ m = radius of cylinder



Let $h(t)$ = height (m) of water level at time t (sec)

Find $\frac{dh}{dt}$

V = volume of circular cylinder $= \pi r^2 h$

here $V = \pi(1\,m^2)h = \pi h\,(m^2)$

$$\frac{dV}{dt} = -3\frac{l}{sec} = \boxed{\frac{-3\times10^{-3}\ m^3}{sec}}$$

$1\text{ L} = 10^{-3}\text{ m}^3$

sec

$1\ \mathrm{L} = 10^{-3}\ \mathrm{m}^3$

$$\frac{dV}{dt} = \frac{d}{dt}(\pi h)\, m^2 =$$

$$\frac{dV}{dt} = \boxed{\pi \frac{dh}{dt}\, m^2}$$

$$\pi \frac{dh}{dt}\, m^2 = -\frac{3\times 10^{-3}\, m^3}{sec}$$

$$\frac{dh}{dt} = -\frac{3\times 10^{-3}\, m^3}{\pi\, m^2\, sec}$$

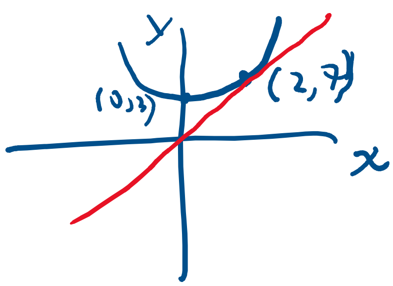
$$\boxed{\frac{dh}{dt} = -\frac{3\times 10^{-3}}{\pi}\,\frac{m}{sec}} \approx \boxed{-.001\,\frac{m}{sec}}$$

-3*10^(-3)/Pi=-0.000954929658551372

6.2.2. $3/(1000\pi)$ meters/second

Your Name MTH 263 quiz 5 write each problem. Calculator required.

1. Use calculus to find the equation of the tangent line to the curve $y = x^2 + 3$ at the point $x = 2$.



$$y(2) = 2^2 + 3 = 7$$

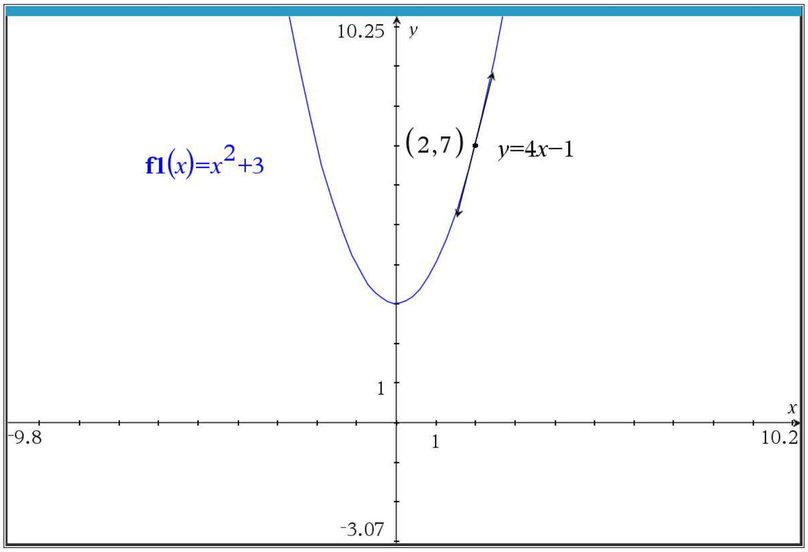
$$f'(x) = \frac{d}{dx}(x^2+3) = 2x$$

$$f'(2) = 2(2) = 4$$

$$\boxed{y - 7 = 4(x-2)} \text{ point-slope}$$

$$y = 4x - 8 + 7 \Rightarrow \boxed{y = 4x - 1}$$

2. Find the equation of the #1 tangent line graphically and sketch a labeled graph from your calculator.



3. Let $xy = x^2 + y^3$. Find $\frac{dy}{dx}$.

$$\frac{d}{dx}(xy) = \frac{d}{dx}(x^2+y^3)$$

$$x\frac{dy}{dx} + y\frac{dx}{dx} = 2x + 3y^2\frac{dy}{dx}$$

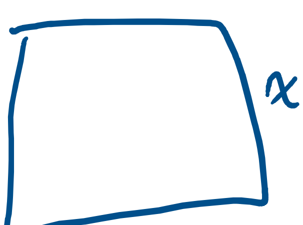
$$x\frac{dy}{dx} + y = 2x + 3y^2\frac{dy}{dx}$$

$$(x-3y^2)\frac{dy}{dx} = 2x - y$$

$$\boxed{\frac{dy}{dx} = \frac{2x-y}{x-3y^2}}$$

4. The side of a square is growing by $\frac{2\ inches}{\text{minute}}$.

How fast is the area changing when the side is 3 inches?



Let $x(t)$ = side of square (in) at time t (min)

Let $A(t)$ = area of square at time t

$\frac{dx}{dt} = 2\frac{\text{in}}{\text{min}}$

Find $\frac{dA}{dt}$ when $x = 3$ in

$A(x) = x^2$

dt $\quad dt$

$$A(x) = x^2$$

$$\frac{dA}{dt} = \frac{d}{dt}(x^2) = 2x\,\frac{dx}{dt}$$

$$= 2(3\text{ in})\left(\frac{2\text{ in}}{\text{min}}\right)$$

$$\boxed{\frac{dA}{dt} = 12\ \frac{\text{in}^2}{\text{min}}}$$