1.5 The Chain Rule
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1.6 Higher Order Derivatives
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2 Derivatives of Common Functions
2.1 Inverse Functions
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Exam 1: 1.1 - 1.6

1.6: 9

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2.5 Flash

What is the mathematical interpretation of one quantity being directly proportional to another?

In mathematics, two quantities are directly proportional if they are related in such a way that their ratio is a constant value. This means that as one quantity increases, the other increases by the same factor, and as one quantity decreases, the other decreases by the same factor.

This relationship is most commonly expressed mathematically in a few key ways:

- **Equation:** If a quantity y is directly proportional to a quantity x, it can be written as the linear equation $y = kx$.
- **Proportionality Symbol:** The relationship can also be expressed using the proportionality symbol, $\propto$, as $y \propto x$.
- **Constant Ratio:** The defining feature is that the ratio of the two quantities is constant. This can be shown as $\frac{y}{x} = k$.

In each of these expressions, k is known as the **constant of proportionality**. This constant represents the fixed rate at which one quantity changes in relation to the other. Graphically, a direct proportion is represented by a straight line that passes through the origin $(0, 0)$. The slope of this line is the constant of proportionality, k.

To help illustrate this, could you share a specific example of direct proportionality that you're currently working with?

9. If an object moves along a straight line such that its position $s(t)$ at time t is directly proportional to t for all t (written as $s \propto t$), then show that the object's acceleration is always 0.

$s = kt$, k = constant

Let $v(t)$ = velocity

$a(t)$ = acceleration

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$$v(t) = \frac{ds}{dt} = \frac{d}{dt}(kt) = k\frac{dt}{dt} = k(1) = k$$

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}(k) = 0$$

$$\therefore \boxed{a(t) = 0}$$

$$\frac{d}{dx}\left(\frac{1}{x^2}\right)$$ use quotient rule

$$= \frac{x^2 \frac{d}{dx}(1) - 1\left(\frac{d}{dx}(x^2)\right)}{(x^2)^2}$$

$$= \frac{x^2(0) - (2x)}{x^4}$$

$$= \frac{-2x}{x^4} = \boxed{-2x^{-3}}$$

$$\frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{d}{dx}(x^{-2})$$

$$= \boxed{-2x^{-3}}$$

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For Exercises 1-18, find the derivative of the given function.

10. $f(x) = \cos\left(\sqrt{x}\right)$

$$f'(x) = -\sin(\sqrt{x})\left(\frac{d}{dx}(\sqrt{x})\right)$$

chain rule

$$f'(x) = -\sin(\sqrt{x})\left(\frac{d}{dx}x^{\frac{1}{2}}\right)$$

$$\boxed{f'(x) = (-\sin\sqrt{x})\left(\frac{1}{2}\right)x^{-\frac{1}{2}}}$$

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3. $f(x) = \sqrt{1 - 2x}$

$$f(x) = (1-2x)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(1-2x)^{-\frac{1}{2}}(-2)$$

$$\boxed{f'(x) = -(1-2x)^{-\frac{1}{2}} = \frac{-1}{\sqrt{1-2x}}}$$

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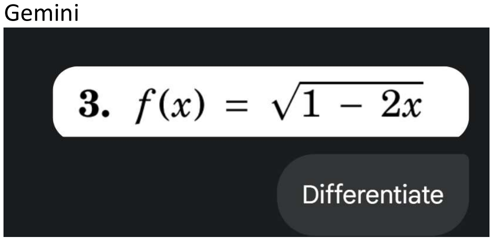

Analysis

Using the chain rule, the derivative of the function $f(x) = \sqrt{1-2x}$ is:

$$f'(x) = \frac{-1}{\sqrt{1-2x}}$$

Here is a breakdown of the steps:

1. Rewrite the function using an exponent: $f(x) = (1-2x)^{1/2}$.
2. Apply the power rule and the chain rule. The derivative of the outer function $(\ldots)^{1/2}$ is $\frac{1}{2}(\ldots)^{-1/2}$. The derivative of the inner function $(1-2x)$ is -2.
3. Multiply the derivatives of the outer and inner functions: $f'(x) = \frac{1}{2}(1-2x)^{-1/2} \cdot (-2)$
4. Simplify the expression: $f'(x) = -1(1-2x)^{-1/2} = \frac{-1}{(1-2x)^{1/2}}$
5. Convert the fractional exponent back to radical form: $f'(x) = \frac{-1}{\sqrt{1-2x}}$

Would you like to try another differentiation problem, perhaps one that requires the product rule or the quotient rule?

1.5: 8

8. $f(x) = \left(\frac{x^2+1}{x-1}\right)^6$

$$f'(x) = 6\left(\frac{x^2+1}{x-1}\right)^5 \frac{d}{dx}\left(\frac{x^2+1}{x-1}\right)$$

use quotient rule

$$= 6\left(\frac{x^2+1}{x-1}\right)^5 \frac{\left[(x-1)\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x-1)\right]}{(x-1)^2}$$

$(x-1)$

$$= 6\left(\frac{x^2+1}{x-1}\right)^5 \left[\frac{(x-1)(2x)-(x^2+1)(1)}{(x-1)^2}\right]$$

$$= \frac{6(x^2+1)^5}{(x-1)^7}\left[2x^2-2x-x^2-1\right]$$

$$\boxed{f'(x) = \frac{6(x^2+1)^5}{(x-1)^7}\left[x^2-2x-1\right]}$$

1.4: 3

For Exercises 1-14, use the rules from this section to find the derivative of the given function.

3. $f(x) = \dfrac{2x^6}{3} - \dfrac{3}{2x^6}$

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$$f'(x) = \left(\frac{2}{3}\right)(6x^5) - \left(\frac{3}{2}\right)\frac{d}{dx}(x^{-6})$$

$$= 4x^5 - \left(\frac{3}{2}\right)(-6)x^{-7}$$

$$= \boxed{4x^5 + \frac{9}{x^7}}$$