

1.4 Derivatives of Sums, Products and Quotients

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1.5 The Chain Rule

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1.6 Higher Order Derivatives

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Exam 1 - Wednesday - 09/17/25

1.1-1.6

1.4:13

A

For Exercises 1-14, use the rules from this section to find the derivative of the given function.

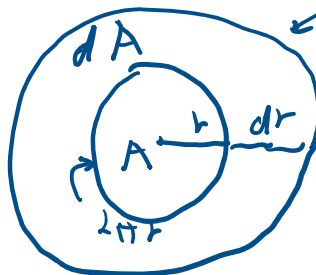
13. $A(r) = \pi r^2$

$$\frac{dA}{dr} = \frac{d}{dr}(\pi r^2) = \pi \frac{d}{dr}(r^2) = \pi(2r^{2-1})$$

↑
constant
multiple
rule

↑
power rule

$$= \boxed{2\pi r}$$

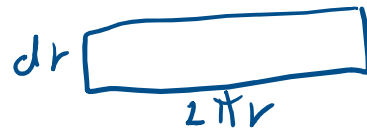


← $\approx 2\pi r$

↘ unroll
the ring

$$A = \pi r^2$$

$$dA = 2\pi r dr$$



$V =$ volume of sphere

$$V = \left(\frac{4}{3}\right)\pi r^3$$

$$\frac{dV}{dr} = \left(\frac{4}{3}\right)\pi(3r^2) = 4\pi r^2$$

$=$ surface area of
a sphere

1.5

Memorize

Chain Rule: If f is a differentiable function of u , and u is a differentiable function of x , then f is a differentiable function of x , and its derivative with respect to x is:

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

Memorize

Chain Rule: If g is a differentiable function of x , and f is a differentiable function on the range of g , then $f \circ g$ is a differentiable function of x , and its derivative with respect to x is:

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

1.5

For Exercises 1-18, find the derivative of the given function.

2. $f(x) = 5(x^3 + x - 1)^4$

$$f'(x) = 20(x^3 + x - 1)^3 (3x^2 + 1)$$

$$(f \circ g)' = [f'(g(x))] g'(x)$$

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

Let $u = x^3 + x - 1$

$f(u) = 5u^4$

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

$$= (20u^3) (3x^2 + 1)$$

$$\boxed{\frac{df}{dx} = 20(x^3 + x - 1)^3 (3x^2 + 1)}$$

Copilot - quick response

2. $f(x) = 5(x^3 + x - 1)^4$

Give the first step in finding the derivative

Great — let's break it down methodically. You're given:

$$f(x) = 5(x^3 + x - 1)^4$$

♦ Step 1: Identify the structure for the chain rule

This is a composite function: a constant multiplied by a power of a polynomial. So we'll apply the **chain rule** to differentiate it.

Let's define:

- Outer function: u^4 , where $u = x^3 + x - 1$
- Inner function: $u = x^3 + x - 1$

Then the derivative becomes:

$$f'(x) = 5 \cdot \frac{d}{dx}[(x^3 + x - 1)^4] = 5 \cdot 4(x^3 + x - 1)^3 \cdot \frac{d}{dx}(x^3 + x - 1)$$

That's the first step: **apply the chain rule to differentiate the outer function and multiply by the derivative of the inner function.**

Want to walk through the next step together?

Sure thing! I'll use standard notation this time. Given:

First, identify the outer and inner functions:

- Outer function: (u^4) , where $(u = x^3 + x - 1)$

- Inner function: $(u = x^3 + x - 1)$

Apply the chain rule:

$\frac{d}{dx}$

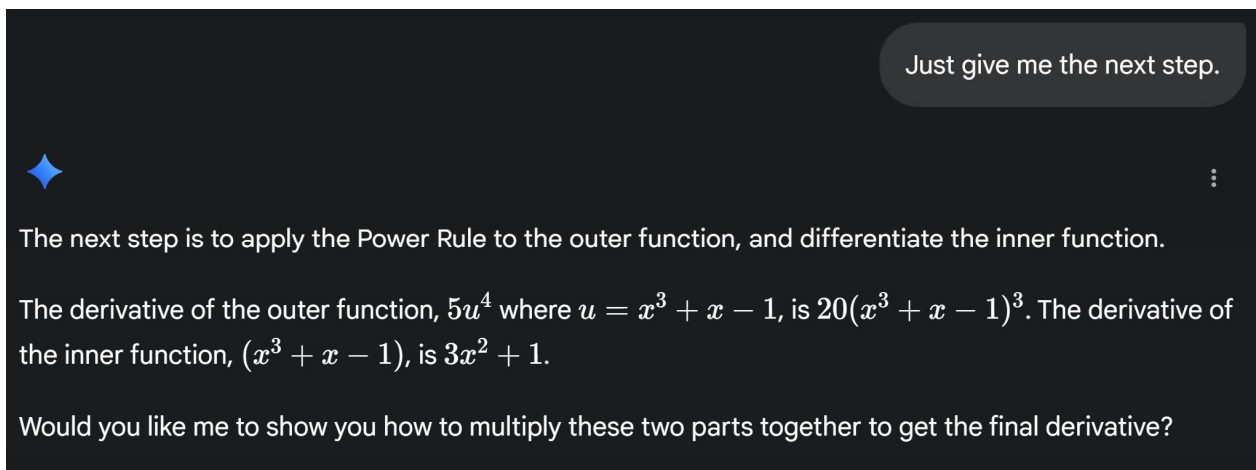
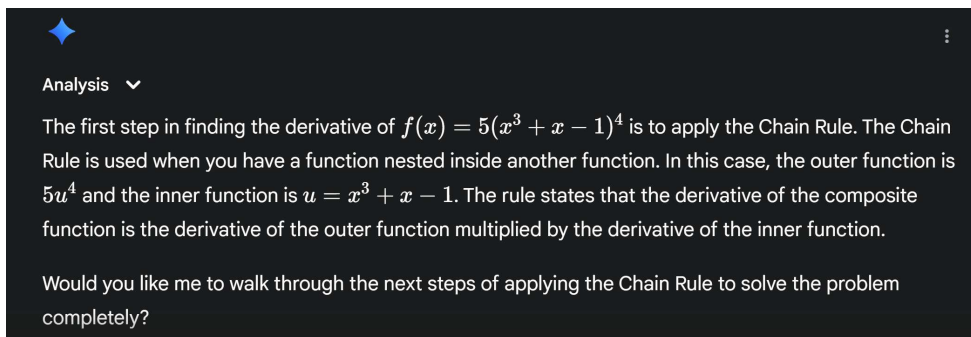
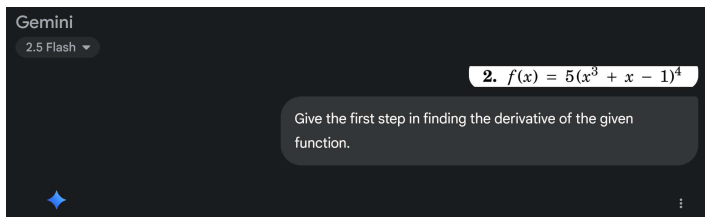
$$f'(x) = 5 \cdot 4(x^3 + x - 1)^3 \cdot \frac{d}{dx}(x^3 + x - 1)$$

So we differentiate u^4 with respect to u , then multiply by the derivative of u with respect to x . Inner function derivative, $3x^2 + 1$, is next.

Ready to continue to the next step?

$\cdot$ dot = $\cdot$, the multiplication symbol
 $\frac{d}{dx}$ is the derivative

$\text{c dot} = \cdot$, the multiplication symbol
 $\text{frac}\{d\}\{dx\} = \frac{d}{dx}$, differential operator



1.4: 5

5. $f(x) = x \sin x$

$$f'(x) = x \frac{d}{dx} (\sin x) + (\sin x) \frac{d}{dx} (x)$$

$$= x \cos x + (\sin x)(1)$$

$$f'(x) = x \cos x + \sin x$$

Notation for the second derivative of $y = f(x)$: The following are all equivalent:

$$f''(x), f^{(2)}(x), \frac{d^2 y}{dx^2}, \frac{d^2}{dx^2}(f(x)), y'', y^{(2)}, \ddot{y}, \ddot{f}(x), \frac{d^2 f}{dx^2}, D^2 f(x)$$

Notation for the n -th derivative of $y = f(x)$: The following are all equivalent:

$$f^{(n)}(x), \frac{d^n y}{dx^n}, \frac{d^n}{dx^n}(f(x)), y^{(n)}, \frac{d^n f}{dx^n}, D^n f(x)$$

Memorize

$$\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$\frac{d^3 y}{dx^3} = \frac{d}{dx} \left(\frac{d^2 y}{dx^2} \right) = \frac{d^2}{dx^2} \left(\frac{dy}{dx} \right)$$

$\vdots$

$$\frac{d^n y}{dx^n} = \frac{d}{dx} \left(\frac{d^{n-1} y}{dx^{n-1}} \right) = \frac{d^{n-1}}{dx^{n-1}} \left(\frac{dy}{dx} \right)$$

Memorize

$s(t)$ = position at time t

$v(t)$ = velocity at time t

$$= \frac{ds}{dt} = s'(t) = \dot{s}(t)$$

$a(t)$ = acceleration at time t

$$= \frac{dv}{dt} = v'(t) = \dot{v}(t)$$

$$= \frac{d}{dt} \left(\frac{ds}{dt} \right) = \frac{d^2 s}{dt^2} = s''(t) = \ddot{s}(t)$$

$$\frac{d^{m+n}}{dx^{m+n}}(f(x)) = \frac{d^m}{dx^m} \left(\frac{d^n}{dx^n}(f(x)) \right) \quad \text{for all integers } m \geq 0 \text{ and } n \geq 0.$$

Recall that the **factorial** $n!$ of an integer $n > 0$ is the product of the integers from 1 to n :

$$n! = 1 \cdot 2 \cdot 3 \cdot \cdots \cdot n$$

$$3! = 3 \cdot 2 \cdot 1 = 6$$

$$\text{def: } 0! = 1$$

$$\frac{d^n}{dx^n}(x^n) = n! \quad \text{for all integers } n \geq 0$$

$$\begin{aligned}\frac{d^{n+1}}{dx^{n+1}}(x^n) &= \frac{d}{dx}\left(\frac{d^n}{dx^n}(x^n)\right) \\ &= \frac{d}{dx}(n!)\end{aligned}$$

915 608 941 463 976 156 518 286 253 697 920 827 223 758 251 185 210 916 864 000 000 000 000 000 000 000 000 = 9.3326 × 10¹⁵⁷

$$h''(x) = (\cos x^2)(2) + 2x(-\sin(x^2))(2x)$$

$$h''(x) = 2 \cos(x^2) - 4x^2 \sin(x^2)$$

$$h''(\sqrt{\pi}) = 2 \cos \pi - 4(\pi) \sin \pi$$

$$h''(\sqrt{\pi}) = (2)(-1) - 4\pi(0)$$

$$h''(\sqrt{\pi}) = -2$$

TI-nspire

$$\frac{d^2}{dx^2}(\sin(x^2))|_{x=\sqrt{\pi}}$$

-2

(4) Find $\lim_{x \rightarrow 3} \frac{x-3}{x-3}$.

$$\lim_{x \rightarrow 3} \frac{x-3}{x-3} = \lim_{x \rightarrow 3} (1) = \boxed{1}$$

(5) Let $f(x) = 6$. Find $\frac{df}{dx}$ using limit of $\frac{\Delta f}{\Delta x}$

$$\frac{df}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{6-6}{h}$$

$$= \lim_{h \rightarrow 0} \frac{0}{h} = \lim_{h \rightarrow 0} (0) = \boxed{0}$$

infinitesimal

$$f(x) = 6$$

$$\frac{df}{dx} = \frac{f(x+dx) - f(x)}{dx}$$

$$= \frac{6-6}{dx} = \frac{0}{dx} = \boxed{0}$$