1-6 Applied Calculus Solutions

Saturday, May 21, 2016 8:12 PM

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Find the degree and leading coefficient of each polynomial

1. $4x^7$

degree 7
leading coefficient 4

3. $5-x^2$

degree 2
leading coefficient -1

5. $-2x^4-3x^2+x-1$

degree 4
leading coefficient -2

Find the vertical and horizontal intercepts of each function.

7. $f(t)=2(t-1)(t+2)(t-3)$

vertical intercept = $f(0)=2(0-1)(0+2)(0-3)=2(-1)(2)(-3)=\boxed{12}$

horizontal intercepts - where $f(t)=0$.

$\boxed{t=1,-2,3}$

9. $g(n)=-2(3n-1)(2n+1)$

vertical intercept = $g(0)=-2\big((3)(0)-1\big)\big((2)(0)+1\big)=$
$-2(-1)(1)=\boxed{2}$

horizontal intercepts - where $g(n)=0$.

$3n-1=0\Rightarrow 3n=1\Rightarrow \boxed{n=\frac{1}{3}}$

$2n+1=0\Rightarrow 2n=-1\Rightarrow \boxed{n=\frac{-1}{2}}$

11. $C(t)=2t^4-8t^3+6t^2$

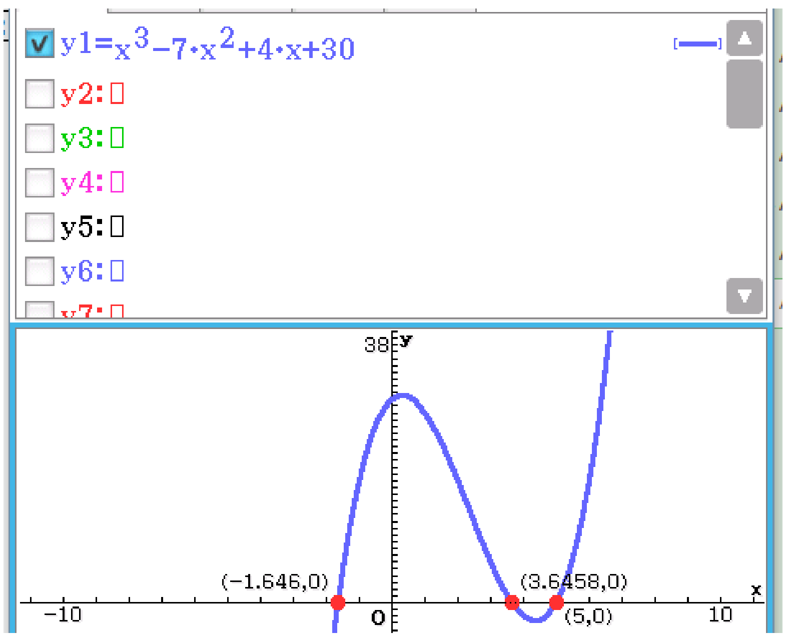
vertical intercept = $C(0) = 2(0^4) - 8(0^3) + 6(0^2) = 0 - 0 + 0 = \boxed{0}$

horizontal intercepts - where $C(t) = 0$

$$2t^4 - 8t^3 + 6t^2 = 0$$
$$\Rightarrow t^4 - 4t^3 + 3t^2 = 0$$
$$\Rightarrow t^2(t^2 - 4t + 3) = 0$$
$$\Rightarrow t^2(t-3)(t-1) = 0$$
$$\Rightarrow \boxed{t = 0,3,1}$$

Use your calculator or other graphing technology to solve graphically for the zeros of the function.

13. $f(x) = x^3 - 7x^2 + 4x + 30$



The zeros are $\boxed{x \approx -1.646,3.458,5}$

Solve each inequality.

15. $(x-3)(x-2)^2 > 0$

Solve the equation $(x-3)(x-2)^2 = 0$.
$x = 2,3$

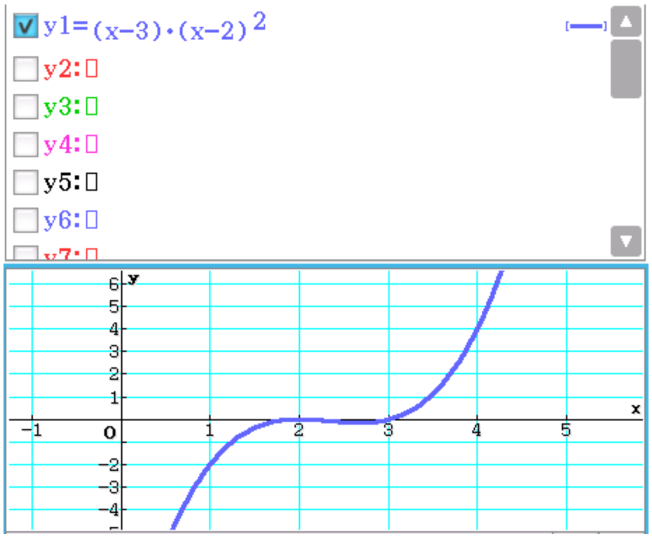
This divides the real number line into intervals: $(-\infty, 2), (2,3), (3, \infty)$.

We choose a convenient test number in each interval. This will determine whether
$(x-3)(x-2)^2$ is positive or negative in that interval.

$$x = 0 \Rightarrow (0-3)(0-2)^2 = (-3)(4) = -12 < 0$$
$$x = 2.5 \Rightarrow (2.5-3)(2.5-2)^2 = (negative)(positive) < 0$$
$$x = 4 \Rightarrow (4-3)(4-2)^2 = (positive)(positive) > 0$$

Therefore, the solution set of the inequality is the interval $(3, \infty)$.

We can check this by graphing the function.



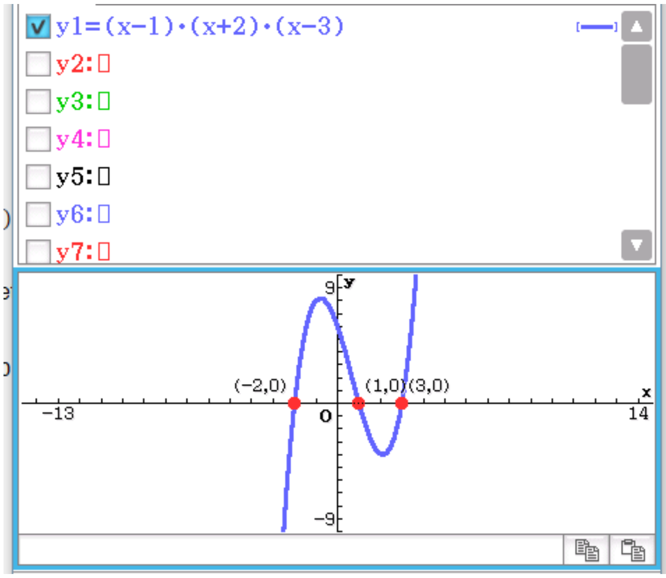
17. $(x-1)(x+2)(x-3) < 0$

Solve the equation $(x-1)(x+2)(x-3) = 0$.
$x = 1, -2, 3$
This divides the real number line into intervals:
$(-\infty, -2), (-2, 1), (1, 3), (3, \infty)$.

We choose a convenient test number in each interval. This will determine whether
$(x-1)(x+2)(x-3)$ is positive or negative in that interval.

$$x = -3 \Rightarrow (-3-1)(-3+2)(-3-3) = (negative)(negative)(ne$$
$$x = 0 \Rightarrow (0-1)(0+2)(0-3) = (negative)(positive)(negati$$
$$x = 2 \Rightarrow (2-1)(2+2)(2-3) = (positive)(positive)(negativ$$
$$x = 4 \Rightarrow\Rightarrow (4-1)(4+2)(4-3) = (positive)(positive)(positi$$

Therefore the solution set of the inequality is: $(-\infty, -2) \cup (1, 3)$.
We can check this by graphing the function.



For each function, find the horizontal intercepts, the vertical intercept, the vertical asymptotes, and the horizontal asymptote.

19. $p(x) = \dfrac{2x-3}{x+4}$

horizontal intercept(s): set $p(x) = 0$ and solve for x.

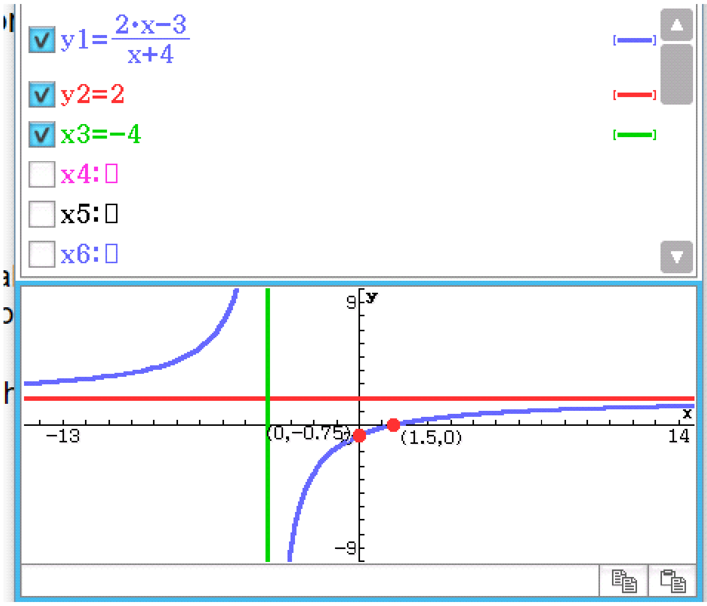
$$\frac{2x-3}{x+4} = 0 \Rightarrow 2x - 3 = 0 \Rightarrow \boxed{x = \frac{3}{2}}$$

vertical intercept = $p(0) = \frac{(2)(0)-3}{0+4} = \frac{-3}{5}$

There is a vertical asymptote at $x = -4$, because this value makes the denominator = 0, and $p(x)$ approaches infinity as x approaches -4.

As x approaches infinity, $p(x)$ approaches $\frac{2x}{x} = 2$, so the horizontal asymptote is the line $y = 2$.

We can check this graphically.



27. A scientist has a beaker containing 20 mL of a solution containing 20% acid. To dilute this, she adds pure water.
    a. Write an equation for the concentration in the beaker after adding n mL of water.
    b. Find the concentration if 10 mL of water has been added.
    c. How many mL of water must be added to obtain a 4% solution?
    d. What is the behavior as $n \to \infty$, and what is the physical significance of this?

(27a) Let $C(n)$ = concentration of solution after n mL of water are

added.

20% of 20mL = $0.20 \times 20mL = 4mL$ of acid.

$$C(n) = \frac{\text{volume of acid}}{\text{volume of solution}} \times 100\% = \frac{4mL}{20mL+nmL} \times 100\%$$

$$\Rightarrow C(n) = \frac{400}{20+n}\%$$

As a check, we see that $C(0) = \frac{400}{20+0}\% = 20\%$

(27b) $C(10) = \frac{400}{20+10}\% = \frac{400}{30}\% \approx 13\%$

(27c) We must solve the equation $C(n) = 4\%$.

$$\Rightarrow \frac{400}{20+n}\% = 4\%$$
$$\Rightarrow \frac{400}{20+n} = 4$$
$$\Rightarrow 400 = 4(20+n)$$
$$\Rightarrow 400 = 80 + 4n$$
$$\Rightarrow 4n = 400 - 80$$
$$\Rightarrow 4n = 320$$
$$\therefore \boxed{n = 80}$$

Therefore, 80mL of water must be added to obtain a 4% acid solution.

(27d) As $n \to \infty, C(n) \to 0$.
This means that the solution becomes so diluted that the acidic
property effectively disappears.

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These solutions were created by Donald R. Goral from exercises in *Applied Calculus, Edition 1*
by Shana Calaway, Dale Hoffman, David Lippman