04-14-26 MTH 261

Section 3.4: Substitution
page 208: 1, 3, 5, 7, 9, 11

Section 3.6: Area, Volume, and Average Value (Area only)
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3.4: 9

For problems 9-12, find an antiderivative of the integrand and use the Fundamental Theorem to evaluate the definite integral.

9. $\int_{-2}^{2} \frac{2x}{1+x^2}\,dx = I$

Let $u = 1+x^2$

$du = 2x\,dx$

$x=-2 \Rightarrow u = 1+(-2)^2 = 1+4 = 5$

$x = 2 \Rightarrow u = 1+2^2 = 1+4 = 5$

$I = \int_5^5 \frac{du}{u} = \ln|u| \Big]_5^5 = \ln|5| - \ln|5|$

$= \boxed{0}$

Alternative presentation

$u = 1+x^2$

$du = 2x\,dx$

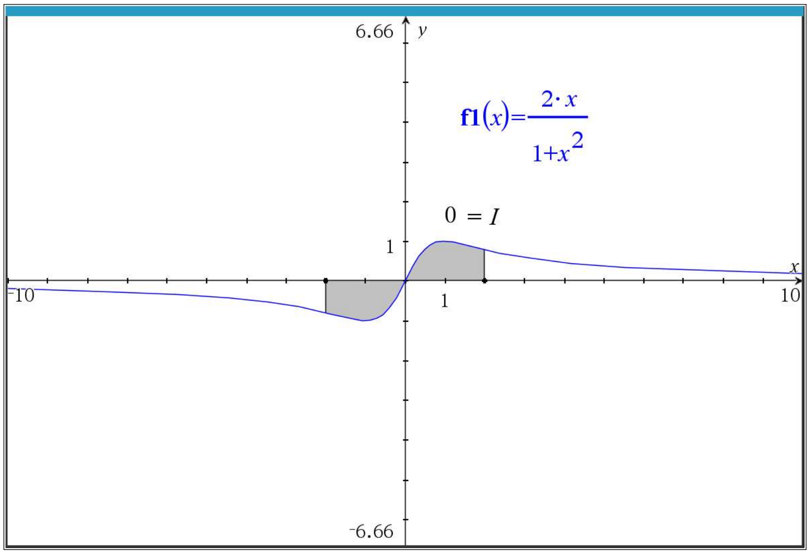
$I = \int_{x=-2}^{2} \frac{du}{u}$

$= \ln|u| \Big]_{x=-2}^{2}$

$= \ln|1+(-2)^2| - \ln|1+2^2|$

$= \ln|5| - \ln|5| = 0$

$= \ln|5| - \ln|5| = 0$



Def $f(x)$ is <u>even</u> if $f(-x) = f(x)$

$f(x)$ is <u>odd</u> if $f(-x) = -f(x)$

let $f(x) = \dfrac{2x}{1+x^2}$

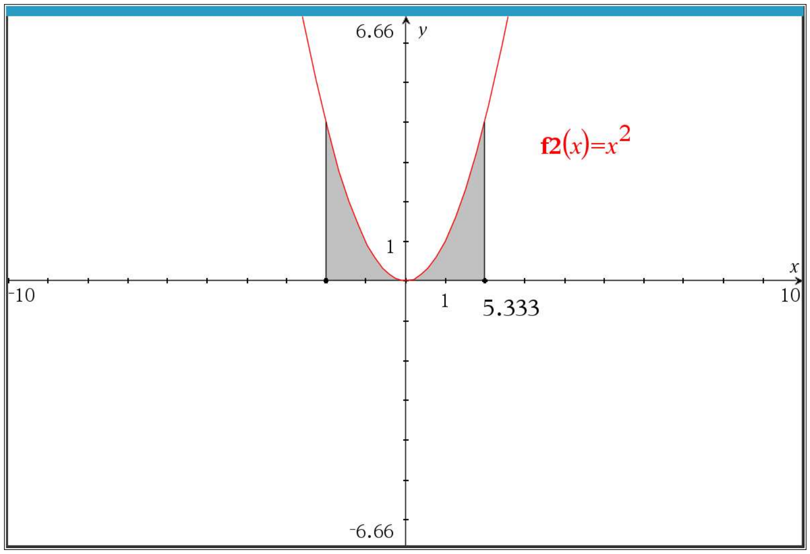
$f(-x) = \dfrac{2(-x)}{1+(-x)^2} = -\dfrac{2x}{1+x^2} = -f(x)$

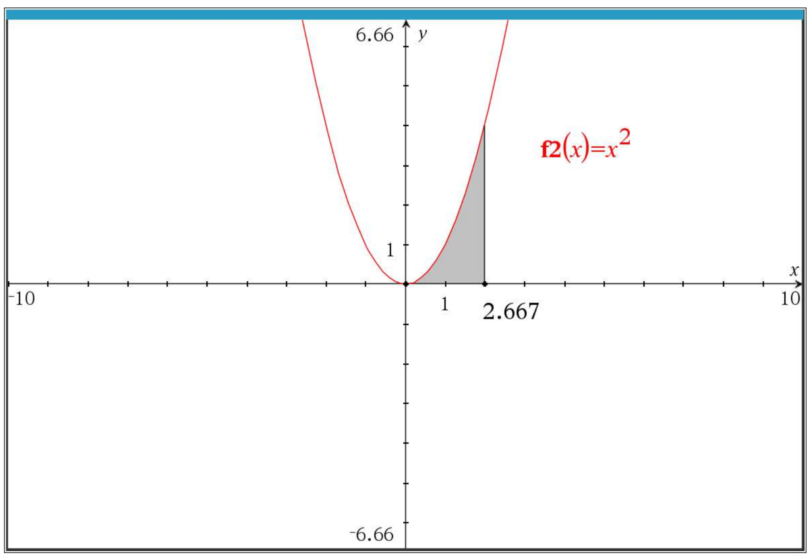
$\therefore f$ is odd

Thm. if f is odd, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$

Thm. if f is even, then $\displaystyle\int_{-a}^{a} f(x)\,dx$

$\displaystyle = 2\int_{0}^{a} f(x)\,dx$





2*2.667=5.334

Prove $\int_a^a f(x)\,dx = 0$

Let $F'(x) = f(x)$

$$\int_a^a f(x)\,dx = F(x)\Big]_a^a$$

$$= F(a) - F(a) = 0$$

3.4: 7

For problems 1-8, find the indicated antiderivative.

du

choose another u

For problems 1-8, find the indicated antiderivative.

7. $\int \frac{dx}{x \ln x} = I$

du

u

Let $u = \ln x$

$\Rightarrow du = \frac{dx}{x}$

$I = \int \frac{du}{u} = \ln|u| + C$

$\boxed{I = \ln|\ln x| + C}$

choose another u

$u = x \ln x \Rightarrow \ln x = \frac{u}{x}$

$\Rightarrow du = \left[x\left(\frac{1}{x}\right) + \ln x\right] dx$

$du = (1 + \ln x)\, dx$

$\frac{du}{1 + \ln x} = dx$

dead end

3.4: 5

5. $\int \sqrt{w+5}\, dw = I$

Let $u = w + 5$

$\Rightarrow du = dw$

$I = \int \sqrt{u}\, du$

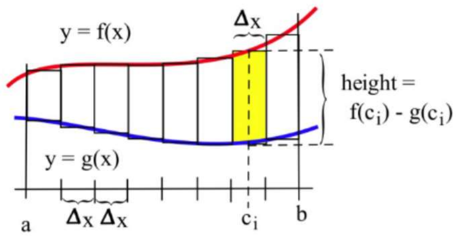
$= \int u^{\frac{1}{2}}\, du$

$= \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{2u^{3/2}}{3} + C$

$\boxed{I = \frac{2}{3}(w+5)^{3/2} + C}$

3.6



The limit of this Riemann sum, as the number of rectangles gets larger and their width gets smaller, is the definite integral $\int_a^b (f(x) - g(x))\,dx$.

The area between two curves f(x) and g(x), where f(x) ≥ g(x), between x = a and x = b is

$$\int_a^b (f(x) - g(x))\,dx$$

The integrand is "top – bottom." Make a graph to be sure which curve is which.

$$\int_a^b (f(x) - g(x))\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

3.6: 5

5. Estimate the area of the island shown