04-09-26 MTH 261

Section 3.3: Antiderivatives of Formulas
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Section 3.4: Substitution
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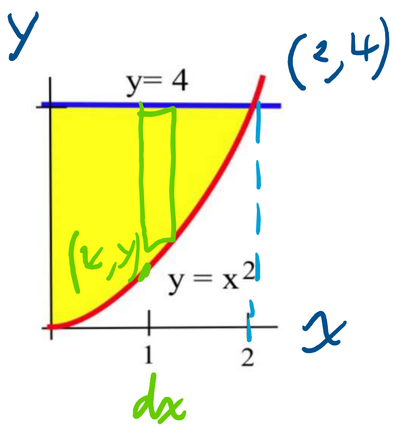
3.3:21

For problems 19 - 21 find the area shown in the figure.



$$\text{Area} = \int_0^2 (4-x^2)\,dx = 4x - \frac{x^3}{3}\Bigg]_0^2$$

$$= 8 - \frac{8}{3} = \boxed{\frac{16}{3}}$$

$=$ area of rectangle

$-$ area under $y = x^2$

$$= 8 - \int_0^2 x^2\,dx$$

$$= 8 - \frac{x^3}{3}\Bigg]_0^2 = 8 - \left[\frac{2^3}{3} - \frac{0^3}{3}\right]$$

$$= 8 - \frac{8}{3} + 0$$

$$= \boxed{\frac{16}{3}}$$

3.4
Memorize

The Substitution Method for Antiderivatives:

The goal is to turn $\int f(g(x))\,dx$ into $\int f(u)\,du$, where $f(u)$ is much less messy than $f(g(x))$.

1. Let u be some part of the integrand. A good first choice is "one step inside the messiest bit."

du

The Substitution Method for Antiderivatives:

The goal is to turn $\int f(g(x))dx$ into $\int f(u)du$, where *f(u)* is much less messy than *f(g(x))*.

1. Let *u* be some part of the integrand. A good first choice is "one step inside the messiest bit."
2. Compute $du = \frac{du}{dx}dx$
3. Translate all your x's into u's everywhere in the integral, including the dx. When you're done, you should have a new integral that is entirely in u. If you have any x's left, then that's an indication that the substitution didn't work or isn't complete; you may need to go back to step 1 and try a different choice for u.
4. Integrate the new u-integral, if possible. If you still can't integrate it, go back to step 1 and try a different choice for u.
5. Finally, substitute back x's for u's everywhere in your answer.

Rule of thumb for u substitution. Look for expressions in parentheses, under a radical, in the denominator, or any messy expression. Then, look if du is obvious.

Evaluate

$$I = \int (x+4) \, dx$$

let $\boxed{u = x+4}$

$$\frac{du}{dx} = \frac{d}{dx}(x+4) = 1$$

$$\Rightarrow du = dx$$

$$I = \int u^{10} \, du$$

$$= \frac{u^{11}}{11} + C$$

$$= \frac{u^{11}}{11} + C$$

$$\boxed{I = \frac{(x+4)^{11}}{11} + C}$$

$$I = \int e^{2x}\,dx$$

Let $u = 2x$

$$\Rightarrow du = 2\,dx \Rightarrow dx = \frac{du}{2}$$

$$I = \int e^{u}\frac{du}{2}$$

$$I = \frac{1}{2}\int e^{u}\,du$$

$$= \left(\frac{1}{2}\right)e^{u} + C$$

$$\boxed{I = \left(\frac{1}{2}\right)e^{2x} + C}$$

check: $\frac{d}{dx}\left(\left(\frac{1}{2}\right)e^{2x} + C\right)$

$$= \left(\frac{1}{2}\right)\left(e^{2x}\right)\frac{d}{dx}(2x) + 0$$

$$= \left(\frac{1}{2}\right)e^{2x}(2)$$

$$= \left(\frac{1}{2}\right) e^{2x} (2)$$

$$= e^{2x} \quad \checkmark$$

$$I = \int e^{2x}\, dx$$

Let $u = e^{2x}$

$$\Rightarrow \frac{du}{dx} = e^{2x} \frac{d}{dx}(2x)$$

$$= e^{2x}(2)$$

$$= 2e^{2x}$$

$$\Rightarrow du = 2\underline{e^{2x}\, dx} \Rightarrow e^{2x} dx = \frac{du}{2}$$

solve for what
we see

$$I = \int \frac{du}{2} = \frac{1}{2}\int du = \left(\frac{1}{2}\right) u + C$$

$$\boxed{I = \frac{e^{2x}}{2} + C}$$

$$I = \int_1^2 (x+3)^4\, dx$$

Let $u = x + 3$

$\Rightarrow du = dx$

$\Rightarrow u = 1 + 3 = 4$

$\Rightarrow \quad du = dx$

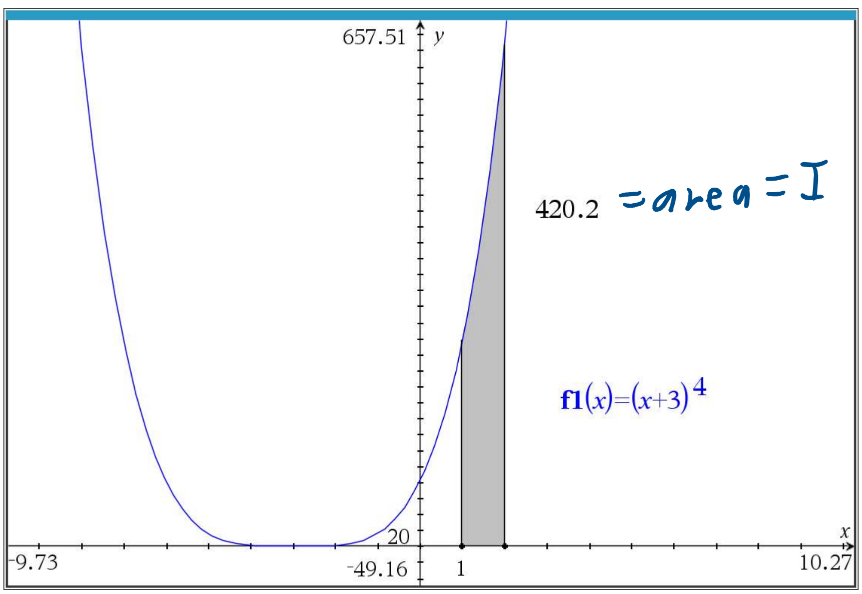
$x = 1 \Rightarrow u = 1 + 3 = 4$

$x = 2 \Rightarrow u = 2 + 3 = 5$

$$I = \int_4^5 u^4 \, du$$

$$= \frac{u^5}{5}\Big]_4^5 = \frac{5^5}{5} - \frac{4^5}{5}$$

$$\boxed{I = \frac{2101}{5} = 420.2}$$



$$I = \int_1^2 (x+3)^4 \, dx$$

Let $u = x + 3$

$\Rightarrow du = dx$

$$I = \int_{x=1}^{2} u^4 \, du$$

$$u^5\Big]^2$$

$x=1$

$$= \frac{u^5}{5}\Big]_{x=1}^{2}$$

$$= \frac{(x+3)^5}{5}\Big]_1^2$$

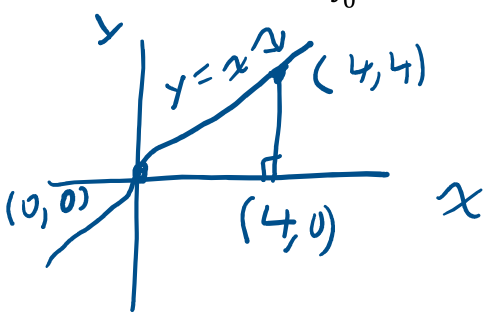
$$= \boxed{\frac{5^5}{5} - \frac{4^5}{5}}$$

Your Name MTH 261 quiz 5 Calculator required. Write each problem.

1. Evaluate $\int x^3 dx$.

$$\boxed{\frac{x^4}{4} + C}$$

2. Use geometry to evaluate $\int_0^4 x dx$. Make a labeled sketch.

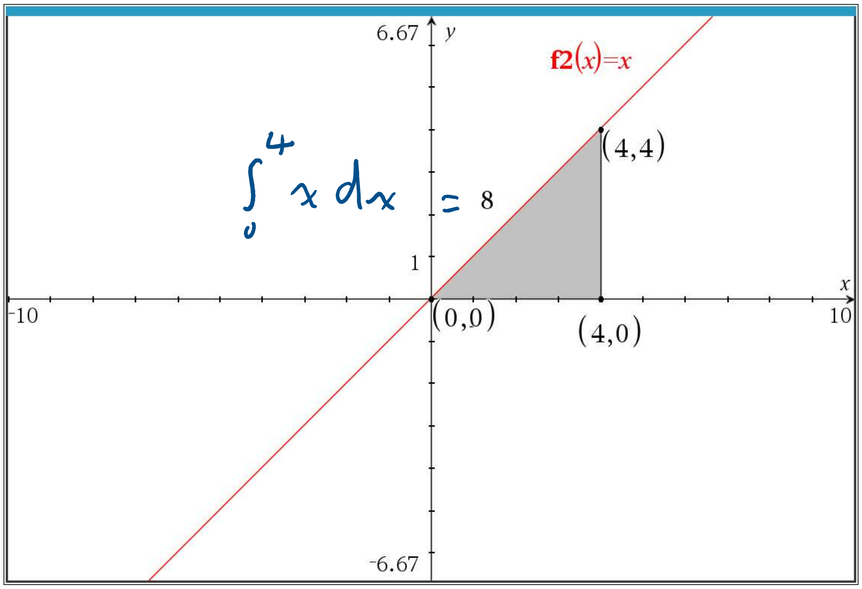


$$\int_0^4 x\,dx = \text{area of } \triangle$$
$$= \left(\frac{1}{2}\right)(4)(4) = \frac{16}{2}$$
$$= \boxed{8}$$

3. Use fundamental theorem of calculus to evaluate $\int_0^4 x dx$.

$$\int_0^4 x\,dx = \frac{x^2}{2}\Big]_0^4 = \frac{4^2}{2} - \frac{0^2}{2} = \frac{16}{2} - 0 = \boxed{8}$$

4. Use your calculator to evaluate $\int_0^4 x dx$. Make a labeled sketch.



5. Evaluate $I = \int \frac{x}{x^2+6} dx$.

let $u = x^2 + 6$

$\Rightarrow du = 2x\,dx \Rightarrow x\,dx = \frac{du}{2}$

$I = \frac{1}{2}\int \frac{du}{u} = \left(\frac{1}{2}\right) \ln|u| + C$

$$\boxed{I = \left(\frac{1}{2}\right) \ln(x^2+6) + C}$$

check: $\frac{d}{dx}\left(\frac{1}{2}\ln(x^2+6) + C\right)$

$= \frac{d}{dx}\left(\frac{1}{2}\ln(x^2+6)\right) + \frac{dC}{dx}$

$= \frac{1}{2}\frac{d}{dx}\left(\ln(x^2+6)\right) + 0$

chain rule

$= \left(\frac{1}{2}\right)\left(\frac{1}{x^2+6}\right)\frac{d}{dx}(x^2+6)$

$= \left(\frac{1}{2}\right)\left(\frac{1}{x^2+6}\right)(2x)$

$=\left(\frac{1}{2}\right)\left(\frac{1}{x^2+6}\right)(2x)$

$= \frac{x}{x^2+6}$ ✓