04-07-26 MTH 261

Section 3.2: The Fundamental Theorem and Antidifferentiation
page 189: 1, 3, 7, 9

Section 3.3: Antiderivatives of Formulas
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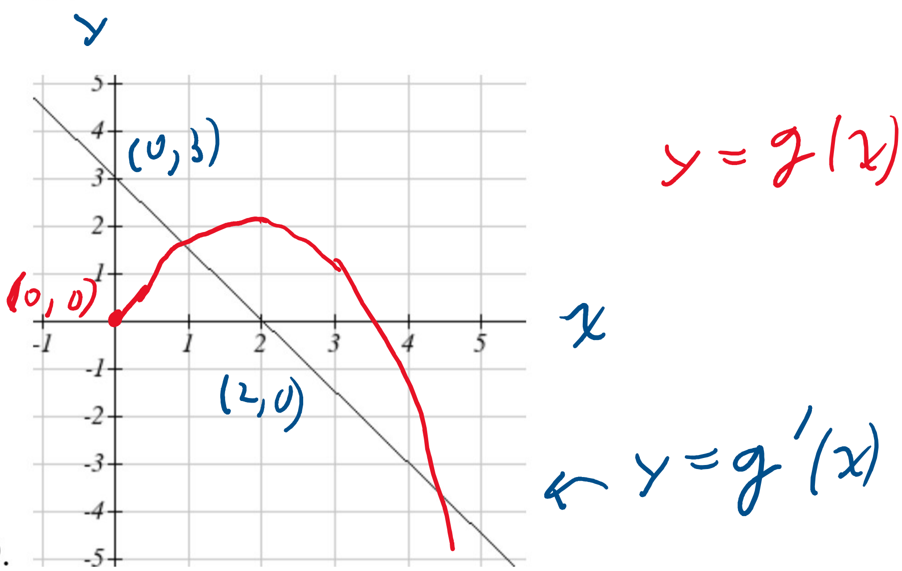
I will add a section on partial derivatives to the homework page

3.2: 9

For problems 9-10, the graph provided shows $g'(x)$. Use it sketch a graph of $g(x)$ that satisfies $g(0) = 0$.

9.



Find equation of the line given by $g'(x)$

$$\text{slope} = \frac{0-3}{2-0} = -\frac{3}{2}$$

$y = mx + b$, m = slope, b = y-intercept

$$\boxed{y = -\frac{3x}{2} + 3}$$

$$g(x) = \int g'(x)\,dx$$

$$= \int \left(-\frac{3x}{2} + 3\right) dx$$

$$= \int -\frac{3x}{2}\,dx + \int 3\,dx$$

$$= -\frac{3}{2}\int x\,dx + 3\int dx$$

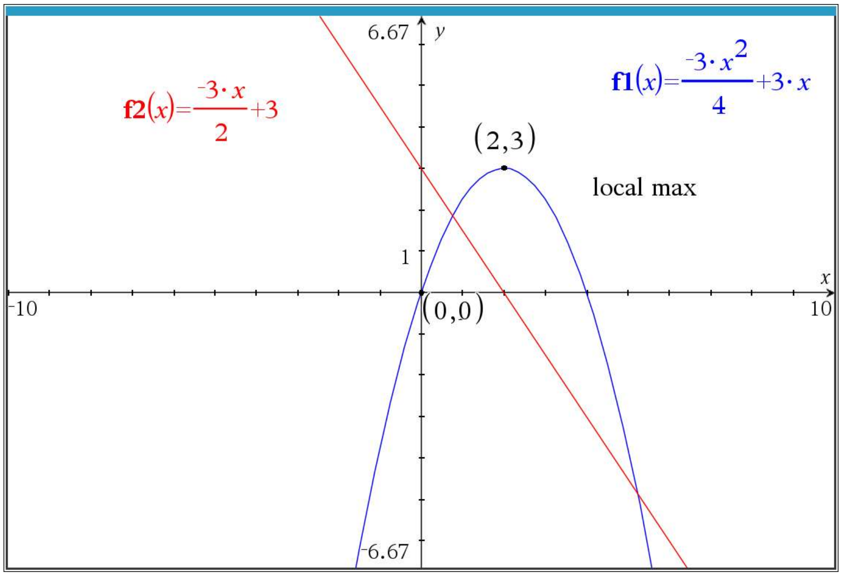
$$g(x) = -\frac{3}{2}\left(\frac{x^2}{2}\right) + 3x + C$$

$$g(x) = -\frac{3x^2}{4} + 3x + C$$

$$g(0) = -\frac{3}{4}(0^2) + 3(0) + C = 0$$

$$\Rightarrow \boxed{C = 0}$$

$$\boxed{g(x) = -\frac{3x^2}{4} + 3x}$$



3.3

Natural Logarithm: $\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$

Why can't we use the power rule here?

Power Rule: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, provided that $n \neq -1$

Special case: $\int k\,dx = kx + C$ (because $k = kx^0$)

$$\int x^{-1}\,dx = \frac{x^{-1+1}}{-1+1} = \frac{x^0}{0} \text{ not defined}$$

Memorize

Antiderivative Rules: Building Blocks

In what follows, f and g are differentiable functions of x and k, n, and C are constants.

(a) **Constant Multiple Rule:** $\int kf(x)\,dx = k\int f(x)dx$

(b) **Sum (or Difference) Rule:** $\int f(x) \pm g(x)dx = \int f(x)dx \pm \int f(x)dx$

(c) **Power Rule:** $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$, provided that n $\neq -1$

Special case: $\int k\,dx = kx + C$ (because $k = kx^0$)

(d) **Exponential Functions:** $\int e^x\,dx = e^x + C$

$\int a^x\,dx = \dfrac{a^x}{\ln a} + C$

(e) **Natural Logarithm:** $\int x^{-1}\,dx = \int \dfrac{1}{x}dx = \ln|x| + C$

Example 3

Find $F(x)$ so that $F'(x) = e^x$ and $F(0) = 10$.

$$F(x) = \int F'(x)\,dx$$

$$= \int e^x dx$$

$$\boxed{F(x) = e^x + C}$$

$$F(0) = e^0 + C = 10$$

$$= 1 + C = 10$$

$$\boxed{C = 9}$$

$$\boxed{F(x) = e^x + 9}$$

Example 4

279 Evaluate $\int_1^3 x\,dx$ in two ways:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F'(x) = f(x)$

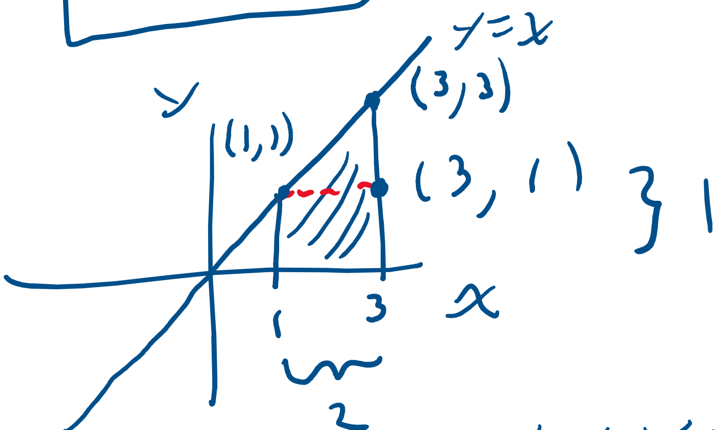
$$\int_1^3 x\,dx = \left.\frac{x^2}{2}\right]_1^3$$

$$= \frac{3^2}{2} - \frac{1^2}{2}$$

$$= \frac{9}{2} - \frac{1}{2}$$

$$= \frac{8}{2}$$

$$\boxed{\int_1^3 x\,dx = 4}$$



Area of rectangle = (2)(1) = 2

Area of $\triangle$ = $\frac{1}{2}(2)(2) = 2$

$$\int_1^3 x\,dx = 2 + 2 = \boxed{4}$$

3.3 Exercises

For problems 1-10, find the indicated antiderivative.

For problems 1-10, find the indicated antiderivative.

4. $\int \pi^2 dw = \pi^2 \int dw = \boxed{\pi^2 w + C}$

10. $\int \frac{t^5 - t^2}{t} dt$ Hint: algebra first, then calculus

$= \int \left(\frac{t^5}{t} - \frac{t^2}{t} \right) dt$

$\int (t^4 - t)\, dt$

$= \boxed{\frac{t^5}{5} - \frac{t^2}{2} + c}$

$t = t^1$

power rule : $\frac{dt}{dt} = 1 \cdot t^{1-1}$

$= 1 \cdot t^0 = 1 \cdot 1$

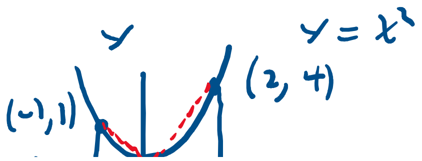
$= \boxed{1}$

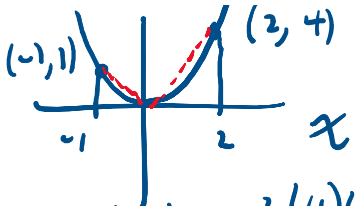
For problems 11-18, find an antiderivative of the integrand and use the Fundamental Theorem to evaluate the definite integral.

12. $\int_{-1}^{2} x^2 \, dx = \left. \frac{x^3}{3} \right]_{-1}^{2}$

$= \frac{2^3}{3} - \frac{(-1)^3}{3}$

$= \frac{8}{3} + \frac{1}{3} = \frac{9}{3} = \boxed{3}$





overestimate $= \frac{1}{2}\big((1)(1)\big) + \frac{1}{2}(2)(4)$

$= \frac{1}{2} + 4 = \boxed{\frac{9}{2} = 4.5}$