04-02-26 MTH 261

Chapter 3: The Integral

PreCalculus Idea – The Area of a Rectangle

Section 3.1: The Definite Integral
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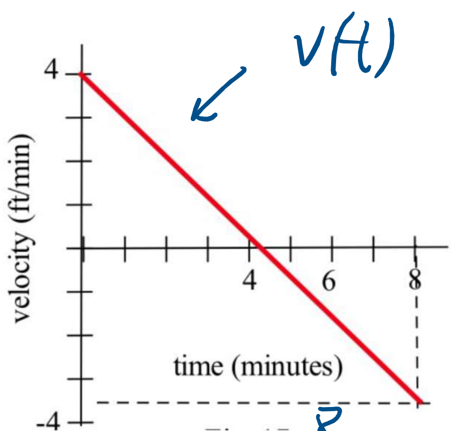
3.1: 21

21. Your velocity along a straight road is shown to the right. How far did you travel in 8 minutes?



The net distance = $\int_0^8 v(t)\,dt$ = area of Δ_1 + area of Δ_2

$$= \left(\frac{1}{2}\right)(4.2)(4) + \frac{(3.8)(-3.5)}{2}$$

$$= \left(\frac{1}{2}\right)(16.8 - 13.3)$$

$$= \frac{1}{2}(3.5) = \boxed{1.75 \text{ ft}}$$

3.8*3.5=13.3

$$2\,\overline{)\,3.50}\quad 1.75$$

This is the distance from the starting point.

If we want the length of the path traversed, then we add $\frac{1}{2}(16.8+13.3) = \frac{1}{2}(30.1) = \boxed{15.05 \text{ ft}}$

$$\begin{array}{r} 16.8 \\ 13.3 \\ \hline 30.1 \end{array}$$

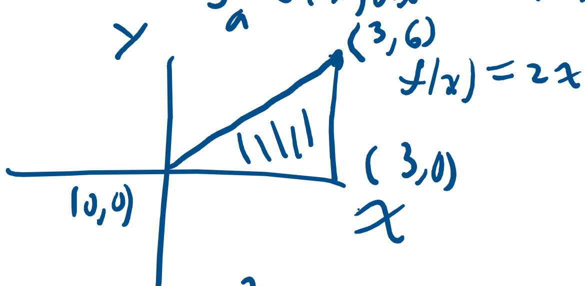
3.2
Memorize

The Fundamental Theorem of Calculus:

$$\int_a^b F'(x)dx = F(b) - F(a)$$

$F'(x) = f(x)$

$\int_a^b f(x)dx = F(b) - F(a)$



Area $\int_0^3 2x\,dx = x^2\Big]_0^3$

$= 3^2 - 0^2 = \boxed{9}$

By geometry $\int_0^3 2x\,dx = \left(\frac{1}{2}\right)(\text{base})(\text{height})$

$= \left(\frac{1}{2}\right)(3)(6) = \boxed{9}$

Memorize

Antiderivatives

An antiderivative of a function f(x) is any function F(x) where F '(x) = f(x).

The antiderivative of a function f(x) is a whole family of functions, written F(x) + C, where F '(x) = f(x) and
C represents any constant.
The antiderivative is also called the **indefinite integral**.

Notation for the antiderivative:
The antiderivative of f is written

$$\int f(x)\,dx$$

This notation resembles the definite integral, because the Fundamental Theorem of Calculus says antiderivatives and definite integrals are intimately related. But in this notation, there are no limits of integration.

The $\int$ symbol is still called an **integral sign**; the dx on the end still must be included; you can still think of $\int$ and dx as left and right parentheses. The function f is still called the **integrand**.

Verb forms:
We **antidifferentiate, or integrate**, or **find the indefinite integral** of a function. This process is called **antidifferentiation** or **integration**.

$$\frac{d}{dx}\left(x^3\right) = 3x^2$$

$$\int 3x^2\,dx = x^3 + C$$

The Fundamental Theorem of Calculus (restated)

$$\int_a^b F'(x)dx = F(b) - F(a)$$

The definite integral of a derivative from a to b gives the net change in the original function.

$$F(b) = F(a) + \int_a^b F'(x)dx$$

The amount we end up is the amount we start with plus the net change in the function.

Memorize

The Fundamental Theorem of Calculus (part 2):

If $A(x) = \int_a^x f(t)dt$, then $A'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$

The Fundamental Theorem of Calculus (part 2):

If $A(x)=\int_a^x f(t)\,dt$, then $A'(x)=\frac{d}{dx}\int_a^x f(t)\,dt=f(x)$

The derivative of the accumulation function is the original function.

Let $A(x) = \int_1^x \ln(t+1)\,dt$

Find $A'(x)$

$$A'(x) = \ln(x+1)$$

Let $f(t) = 2t$

$A(x) = \int_1^x f(t)\,dt$

Find $A'(x)$

$$\int_1^x f(t)\,dt = \int_1^x 2t\,dt$$

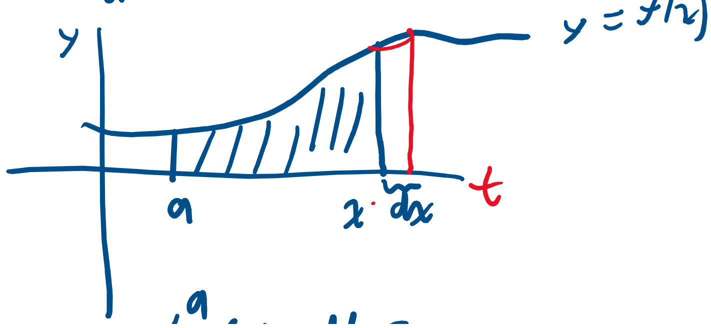
$$= t^2\Big]_1^x$$

$$= x^2 - 1^2$$

$$\boxed{\int_1^x 2t\,dt = x^2-1}$$

$$\frac{d}{dx}(x^2-1) = \boxed{2x}$$

$A(x) = \int_a^x f(t)\,dt = F(x) - F(a)$



$$\int_a^a f(t)\,dt = 0$$

$$A(x+dx) = A(x) + A(dx)$$
$$= A(x) + f(x)\,dx$$
$$\Delta A = A(x+dx) - A(x) = A(dx)$$
$$= f(x)\,dx$$
$$\Rightarrow \frac{dA}{dx} = f(x)$$

3.2 Exercises

In problems 1 – 5, verify that $F(x)$ is an antiderivative of the integrand $f(x)$ and use Part 2 of the Fundamental Theorem to evaluate the definite integrals.

2. $\int_1^4 3x^2\,dx$, $F(x) = x^3 + 2$

$$f(x) = 3x^2$$
$$F'(x) = \frac{d}{dx}(x^3 + 2)$$
$$= \frac{d}{dx}(x^3) + \frac{d}{dx}(2)$$
$$= 3x^2 + 0$$
$$= 3x^2 \quad \checkmark$$

The Fundamental Theorem of Calculus:

$$\int_a^b F'(x)dx = F(b) - F(a)$$

$$\int_1^4 3x^2\,dx = x^3 + 2\Big]_1^4$$

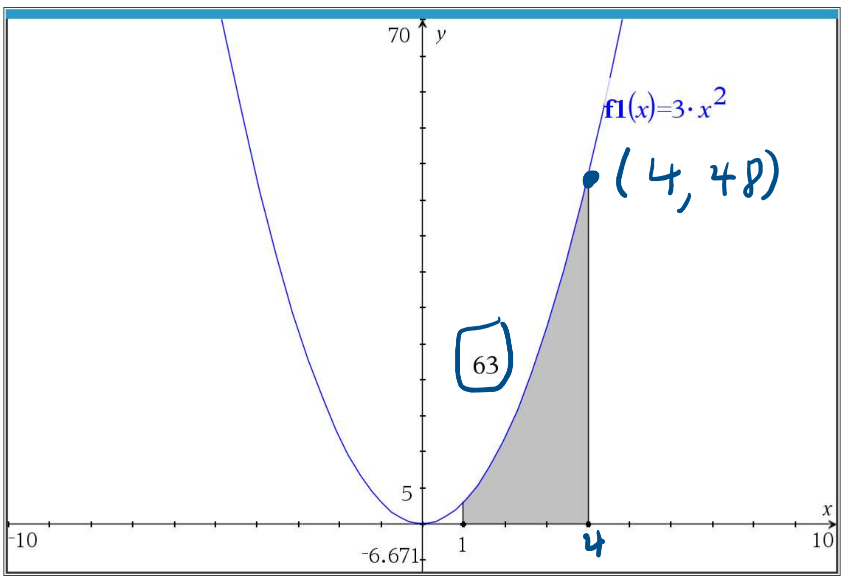
$$\int_1^4 3x^2\,dx = x^3+2\Big]_1^4$$

$$= (4^3+2)-(1^3+2)$$

$$= 4^3-1^3$$

$$= 64-1$$

$$= \boxed{63}$$



For a definite integral do not write the arbitrary constant C.