03-26-26 MTH 261

Section 2.10: Other Applications
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Section 2.11: Implicit Differentiation and Related Rates
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2.10: 7

7. The demand function for Alicia's oven mitts is given by $q = -8p + 80$ (q is the number of oven mitts, p is the price in dollars). Find the elasticity of demand when p = \$7.50. Will revenue increase if Alicia raises her price from \$7.50?

Elasticity of Demand
Given a demand function that gives q in terms of p,

The elasticity of demand is $E = \left|\frac{p}{q} \cdot \frac{dq}{dp}\right|$

(Note that since demand is a decreasing function of p, the derivative is negative. That's why we have the absolute values – so E will always be positive.)

If **E < 1,** we say demand is **inelastic.** In this case, raising prices increases revenue.
If **E > 1**, we say demand is **elastic.** In this case, raising prices decreases revenue.
If **E = 1**, we say demand is **unitary**. E = 1 at critical points of the revenue function.

Interpretation of elasticity:
If the price increases by 1%, the demand will decrease by **E**%.

To calculate E, we need values for p, q, and $\frac{dq}{dp}$.

$\boxed{p = 7.50}$

$q = -8(7.50) + 80$

$q = -60.00 + 80$

$\boxed{q = 20 \text{ mitts}}$

$q = -8p + 80$

$\frac{dq}{dp} = \frac{d}{dp}(-8p) + \frac{d}{dp}(80)$

$\frac{dq}{dp} = -8\frac{dp}{dp} + 0 \quad | \quad d(p) = d(p')$

$$\frac{dq}{dp} = -8\frac{dp}{dp} + 0 \qquad \frac{d}{dp}(p) = \frac{d(p^1)}{dp}$$

$$\frac{dq}{dp} = (-8)(1) + 0 \qquad = 1p^{1-1}$$

$$\boxed{\frac{dq}{dp} = -8} \qquad = 1 \cdot p^0 = 1 \cdot 1 = 1$$

$$E = \left| \left(\frac{7.5}{20}\right)(-8) \right|$$

$$E = \frac{(7.5)(\cancel{8}^{\,2})}{\cancel{20}\,5}$$

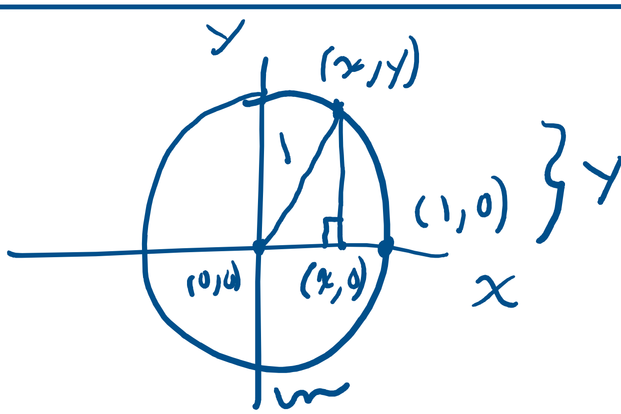
$$E = \frac{15.0}{5}$$

$$\boxed{E = 3 > 1}$$

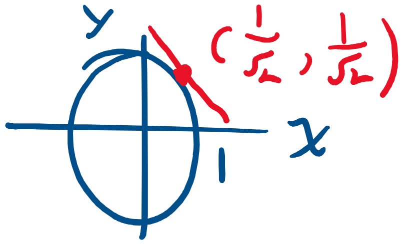
$\therefore$ demand is elastic

No. Revenue will <u>not</u> increase,
because the revenue will decrease.

2.11



$$\overset{x}{\boxed{x^2+y^2=1}}$$ equation of unit circle



$$\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2$$
$$=\frac{1}{2}+\frac{1}{2}=1$$

Find $\frac{dy}{dx}$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$$y^2=1-x^2$$
$$y=\pm\sqrt{1-x^2}$$

choose $y=\sqrt{1-x^2}$

$$y=(1-x^2)^{\frac{1}{2}}$$
$$\frac{dy}{dx}=\left(\frac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)$$
$$\boxed{\frac{dy}{dx}=\frac{-x}{\sqrt{1-x^2}}}$$

$$\frac{dy}{dx}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)=\frac{-\frac{1}{\sqrt{2}}}{\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}}$$
$$=\frac{-\frac{1}{\sqrt{2}}}{\sqrt{1-\frac{1}{2}}}=\frac{-\frac{1}{\sqrt{2}}}{\sqrt{\frac{1}{2}}}=\frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}=-1$$

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implicit differentiation

$\sqrt{2}$

implicit differentiation

$$x^2 + y^2 = 1$$

Differentiate each term with respect to x.
Then, solve algebraically for $\frac{dy}{dx}$.

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(1)$$

$$2x + (2y)\frac{dy}{dx} = 0$$

$$x + y\frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

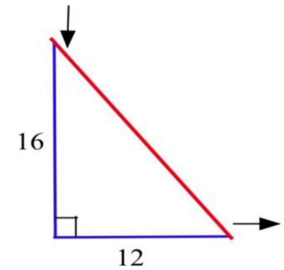
$$\frac{dy}{dx}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = -1$$

Memorize

Related Rates
When working with a related rates problem,
1. Identify the quantities that are changing, and assign them variables
2. Find an equation that relates those quantities
3. Differentiate both sides of that equation with respect to time
4. Plug in any known values for the variables or rates of change
5. Solve for the desired rate.

2.11

18. The 12 inch base of a right triangle is growing at 3 inches per hour, and the 16 inch height is shrinking at 3 inches per hour.
   (a) Is the area increasing or decreasing?
   (b) Is the perimeter increasing or decreasing?
   (c) Is the hypotenuse increasing or decreasing?



Let x = base (in)
Let y = height (in)
Let t = time (hr)

$$\boxed{\frac{dx}{dt} = \frac{3\text{ in}}{\text{hr}}}$$

$$\boxed{\frac{dy}{dt} = -\frac{3\text{ in}}{\text{hr}}}$$

(a) Let $A(x, y)$ = area of $\triangle$

$$A = \left(\frac{1}{2}\right)(xy)$$

$$\frac{dA}{dt} = \left(\frac{1}{2}\right)\frac{d}{dt}(xy)$$

$$\boxed{\frac{dA}{dt} = \left(\frac{1}{2}\right)\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right)}$$

$$\frac{dA}{dt} = \left(\frac{1}{2}\right)\left((12\text{ in})\left(-\frac{3\text{ in}}{\text{hr}}\right) + (16\text{ in})\left(\frac{3\text{ in}}{\text{hr}}\right)\right)$$

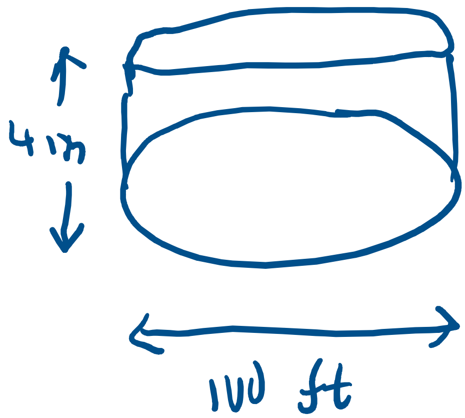
$$\frac{dA}{dt} = -18\frac{\text{in}^2}{\text{hr}} + 24\frac{\text{in}^2}{\text{hr}}$$

dt $\qquad hr$ $\qquad hr$

$$\boxed{\frac{dA}{dt} = 6\ \frac{in^2}{hr} > 0}$$

$\therefore$ Area is increasing

22. An oil tanker in Puget Sound has sprung a leak, and a circular oil slick is forming. The oil slick is 4 inches thick everywhere, is 100 feet in diameter, and the diameter is increasing at 12 feet per hour. Your job, as the Coast Guard commander or the tanker's captain, is to determine how fast the oil is leaking from the tanker.



Let $V(t)$ = volume (ft^3) of leaked oil at time t (hr)

Goal: find $\frac{dV}{dt}$

r = radius = $\frac{\text{diameter}}{2}$

$$\frac{dr}{dt} = \left(\frac{1}{2}\right) \frac{d(\text{diameter})}{dt}$$

$$\boxed{\frac{dr}{dt} = \left(\frac{1}{2}\right)\left(12\ \frac{ft}{hr}\right)}$$

$$\boxed{\frac{dr}{dt} = 6\ \frac{ft}{hr}}$$

dt   hr

$$V = \pi r^2 (4\,in) = \pi r^2 \left(\frac{1}{3}\,ft\right)$$

$$\boxed{V = \frac{\pi r^2}{3}}$$

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{\pi r^2}{3}\right) = \frac{\pi}{3}\frac{d}{dt}\left(r^2\right)$$

$$= \left(\frac{2\pi}{3}\right)\left(r\frac{dr}{dt}\right)$$

$$\boxed{\frac{dV}{dt} = \frac{2\pi}{3}\, r\, \frac{dr}{dt}}$$

$$r = \frac{100\,ft}{2} = 50\,ft$$

$$\frac{dV}{dt} = \left(\frac{2\pi}{3}\right)(50\,ft)\left(6\,\frac{ft}{hr}\right)^{ft}$$

$$\boxed{\frac{dV}{dt} = 200\pi\,\frac{ft^3}{hr}}$$