03-10-26 MTH 261

Section 2.9: Applied Optimization
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Exam 2, Thursday, 03/12/26
2.6-2.9

Before class notes
2.7

In problems 3 – 8, find all of the critical points and local maximums and minimums of each function.

5. $f(x) = x^3 - 6x^2 + 5$

$\boxed{f'(x) = 3x^2 - 12x}$ defined for all x

set $f'(x) = 0$, solve for x

$3x^2 - 12x = 0$

$\frac{3x^2}{3} - \frac{12x}{3} = \frac{0}{3}$

$x^2 - 4x = 0$

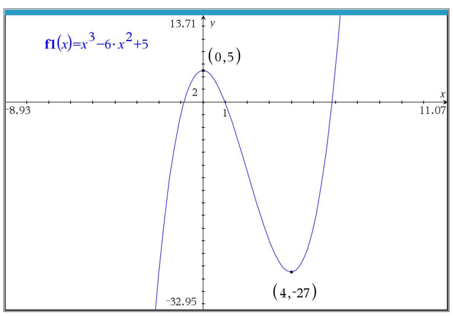
$x(x - 4) = 0$

$\boxed{x = 0, 4}$ critical points

$f''(x) = 6x - 12$

$f''(0) = 6(0) - 12 = -12 < 0$ local max

$f''(4) = 6(4) - 12 = 12 > 0$ local min



2.7

In problems 9 – 16, find all critical points and global extremes of each function on the given intervals.

13. $f(x) = x^2 - 6x + 5$ on $[-2, 5]$.

$f'(x) = 2x - 6$ defined for all $x \in [-2, 5]$

element of a set

$2x - 6 = 0$

$2x = 6$

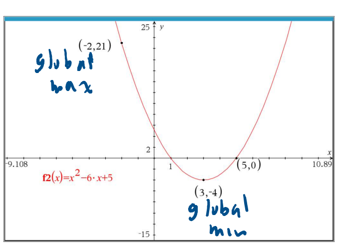
$\boxed{x = 3}$ critical point

$f(3) = 3^2 - 6(3) + 5 = -4$

$\boxed{\text{critical point } (3, -4)}$ global min

$f(-2) = (-2)^2 - 6(-2) + 5 = 21$ $\boxed{\text{global max } (-2, 21)}$

$f(5) = (5)^2 - 6(5) + 5 = 0$



d (k·f(x)) k = constant

$$\frac{d}{dx}\left(k\cdot f(x)\right) \qquad k = \text{constant}$$

$$k\frac{d}{dx}f(x) = k f'(x)$$

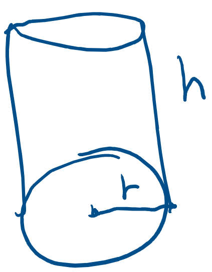
$$\frac{d}{dx}\left(3x^2\right) = 3\frac{d}{dx}\left(x^2\right)$$

$$= 6x$$

2.9 :7

$2¢/in^2 \quad 5¢/in^2 \quad 3¢/in^2$

7. (a) Determine the dimensions of the least expensive cylindrical can which will hold 100 cubic inches if the materials cost 2¢, 5¢ and 3¢ respectively for the top, bottom and sides.
   (b) How do the dimensions of the least expensive can change if the bottom material costs more than 5¢ per square inch?



Let h = height of can

r = radius of can

geometry supplied

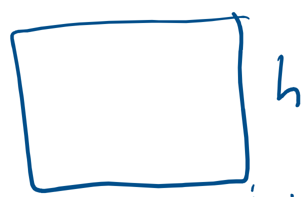
A_b = Area of bottom

A_t = Area of top

A_s = Area of side

side

$$\boxed{\begin{aligned} A_b &= \pi r^2 \\ A_t &= \pi r^2 \\ A_s &= 2\pi r \end{aligned}}$$



$2\pi r$ = circumference of can

Let V = volume

$$\boxed{V = \pi r^2 h = 100\,in^3} \Rightarrow \boxed{h = \frac{100\,in^3}{\pi r^2}}$$

Minimize cost

Let $c(r,h)$ = cost of top
+ cost of bottom
+ cost of side

$$= A_t\left(\frac{cost}{in^2}\right) + A_b\left(\frac{cost}{in^2}\right) + A_s\left(\frac{cost}{in^2}\right)$$

$= A_t \left(\frac{c/in}{in^2}\right) + A_b \left(\frac{c/in}{in^2}\right) + A_s \left(\frac{c/in}{in^2}\right)$

$C(r,h) = \pi r^2 \left(\frac{2¢}{in^2}\right) + \pi r^2 \left(\frac{5¢}{in^2}\right) + 2\pi r h \left(\frac{3¢}{in^2}\right)$

$C(r) = \frac{2\pi r^2 ¢}{in^2} + \frac{5\pi r^2 ¢}{in^2} + 2\pi r \left(\frac{100\ in^3}{\pi r^2}\right)\left(\frac{3¢}{in^2}\right)$

$C(r) = \frac{7\pi r^2 ¢}{in^2} + \frac{600\pi r}{\pi r^2} \frac{¢}{in^2}$

$C(r) = \left(7\pi r^2 + \frac{600}{r}\right) \frac{¢}{in^2}$

$C'(r) = 14\pi r + \frac{d}{dr}\left(600 r^{-1}\right)$

$C'(r) = 14\pi r - 600 r^{-2} = 0 \qquad r \neq 0$

$14\pi r - \frac{600}{r^2} = 0$

$14\pi r^3 - 600 = 0$

$r^3 = \frac{600}{14\pi}$

$r = \sqrt[3]{\frac{600}{14\pi}}$

$r = \sqrt[3]{\frac{300}{7\pi}} \approx \boxed{5.1\ in}$

$\sqrt[3]{300/7\pi}$

5.12535985

$h = \frac{100}{\pi(5.1)^2} \approx \boxed{1.21 n}$

100/(π×5.1^2)

1.223798101

$\pi(5.1)^2(1.21)$

Pi*(5.1)^2*1.21=98.87251815304332

2.9: 17

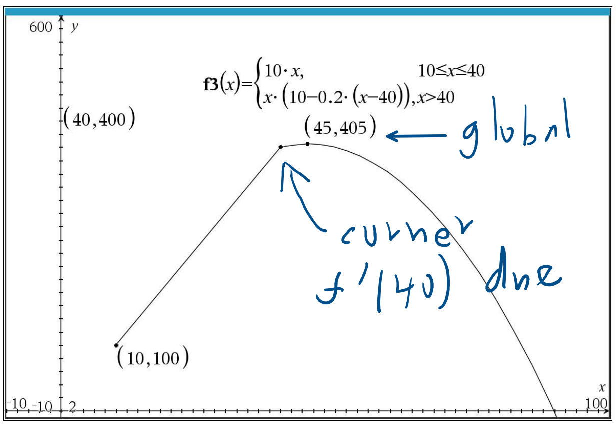
17. In the planning of a taco restaurant, we estimate that if there is seating for between 10 and 40 people, the daily profit will be \$10 per seat. However, if the seating capacity is more than 40 places, the daily profit per seat will be decreased by \$0.20 per seat. What should the seating capacity be in order to maximize the taco restaurant's total profit?

Let $P(n)$ = profit for seating n people places, the daily profit per seat will be decreased by $0.20 per seat. What should the seating capacity be in order to maximize the taco restaurant's total profit?

Let $P(x)$ = profit for seating x people

Find x to maximize $P(x)$

$$P(x) = \begin{cases} 10x & \text{for } 10 \le x \le 40 \\ x(10 - .2(x-40)) & x > 40 \end{cases}$$

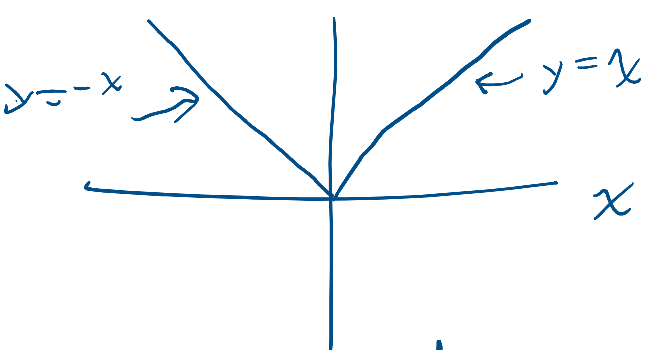


After class notes

Let $f(x) = |x|$

continuous for all x

$$\lim_{x \to 0} |x| = 0$$



If $x > 0$, $\frac{d}{dx}|x| = \frac{d}{dx}(x) = 1$

If $x < 0$, $\frac{d}{dx}|x| = \frac{d}{dx}(-x) = -1$

$\frac{d}{dx}|x|$ $= \lim \frac{|x+h| - |x|}{} = \lim \frac{|0+h| - |0|}{}$

$$\frac{d}{dx}|x|\Big|_{x=0} = \lim_{h\to 0}\frac{|x+h|-|x|}{h} = \lim_{h\to 0}\frac{|0+h|-|0|}{h}$$

$$= \lim_{h\to 0}\frac{|h|}{h}$$

$$\lim_{h\to 0^-}\frac{|h|}{h} = \frac{+}{-} = -1$$

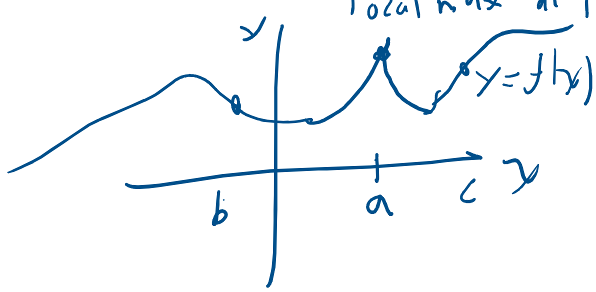
$$\lim_{h\to 0^+}\frac{|h|}{h} = \frac{+}{+} = 1$$

$$-1 \neq 1$$

$$\therefore \lim_{h\to 0}\frac{|0+h|-|0|}{h} \text{ dne}$$

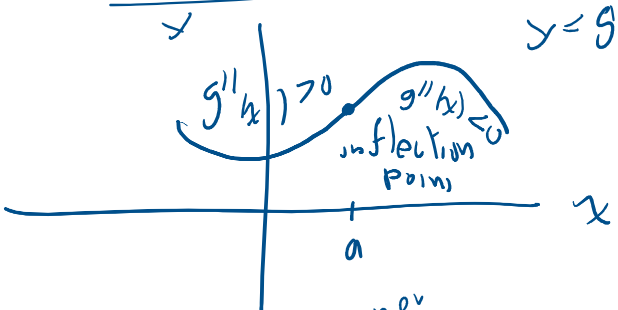
$$\therefore \frac{d}{dx}|x|\Big|_{x=0} \text{ dne}$$

local max at $(a, f(a))$



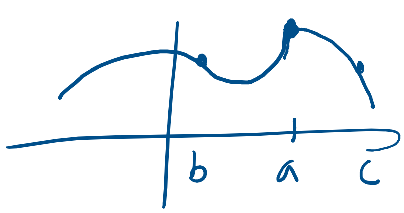
but $\frac{df}{dx}\Big|_{x=a}$ dne

concave up on (b, a) and on (a, c)



$y = g(x)$, $g''(a) = 0$

corner



$y = K(x)$

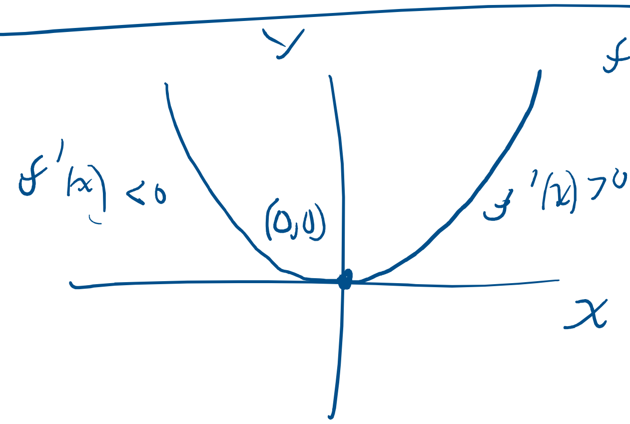
on (b, a) $K''(x) > 0$

on (b, a), $K''(x) > 0$
concave up

(a, c) $K'' < 0$
concave down

$K'(a)$ dne
$K(a)$ is well-defined

The point $(a, K(a))$ is an inflection point



$f(x) = x^2$

$f'(x) = 2x$ defined for x

$2x = 0$
$\Rightarrow \boxed{x = 0}$ critical number

$f(0) = 0^2 = 0$
critical point $(0,0)$

2nd derivative test
$f''(x) = 2 > 0$ all x
$\therefore$ $f(x)$ is concave up on $(-\infty, \infty)$
and local min at $(0,0)$

1st derivative test

$f'(x) > 0$	$f'(x) < 0$
$2x > 0$	$2x < 0$
$\Rightarrow x > 0$	$x < 0$

Let $f(x) = 2x^3 - 4x^2 + 1$ Look for inflection points

$f'(x) = 6x^2 - 8x$

Let $f(x)$ ...

$f'(x) = 6x^2 - 8x$

$f''(x) = 12x - 8$

$12x - 8 = 0$

$3x - 2 = 0$

$3x = 2$

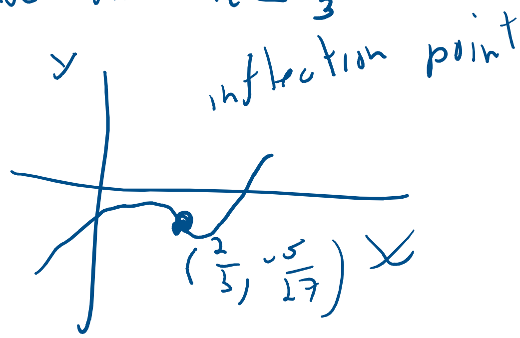
$\boxed{x = \frac{2}{3}}$ candidate for inflection point

concave up $f''(x) > 0$

$12x - 8 > 0$

$\boxed{x > \frac{2}{3}}$

concave down $x < \frac{2}{3}$

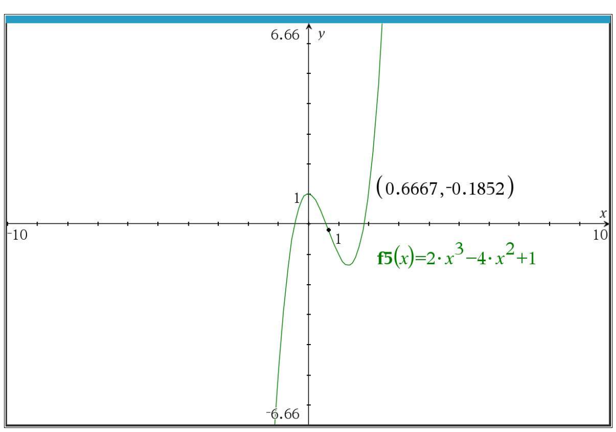


$f\left(\frac{2}{3}\right) = 2\left(\frac{2}{3}\right)^3 - 4\left(\frac{2}{3}\right)^2 + 1$

$= \frac{16}{27} - \frac{16}{9} + 1$

$= \frac{16}{27} - \frac{48}{27} + \frac{27}{27}$

$= -\frac{5}{27}$



-5/27=-0.1852

$f(x) = 3$

$f'(x) = 0$

... all x

$f'(x) = 0$

$f''(x) = 0$ all x

concave up $f''(x) > 0$

$\Leftrightarrow$ $0 > 0$ impossible

concave down $f''(x) < 0$

$0 < 0$ impossible