03-05-26 MTH 261

Section 2.8: Curve Sketching
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Section 2.9: Applied Optimization
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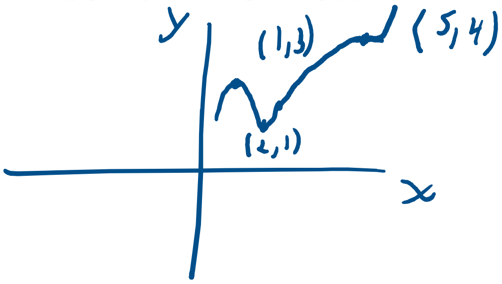
Exam 2, Thursday, 03/12/26
2.6-2.9
2.8: 1

2.8 Exercises

1. Sketch the graph of a continuous function f so that
   (a) $f(1) = 3$, $f'(1) = 0$, and the point $(1,3)$ is a local maximum of f.
   (b) $f(2) = 1$, $f'(2) = 0$, and the point $(2,1)$ is a local minimum of f.
   (c) $f(5) = 4$, $f'(5) = 0$, and the point $(5,4)$ is not a local minimum or maximum of f.

(1b)



2.8: 13

In problems 10 – 14, use information from the derivatives of each function to help you graph the function. Find all local maximums and minimums of each function.

13. $r(t) = \dfrac{2}{t^2 + 1}$

$$r(t) = 2(t^2+1)^{-1}$$

$$r'(t) = -2(t^2+1)^{-2}(2t)$$

$$\boxed{r'(t) = -4t(t^2+1)^{-2}}$$

$$r''(t) = -4t(-2)(t^2+1)^{-3}(2t) + (t^2+1)^{-2}(-4)$$

$$\boxed{r''(t) = \frac{16t^2}{(t^2+1)^3} - \frac{4}{(t^2+1)^2}}$$

critical point

$$\frac{-4t}{(t^2+1)^2} = 0$$

$$\frac{-4t}{(t^2+1)^2} = 0$$

$$-4t = 0$$

$$\boxed{t = 0}$$

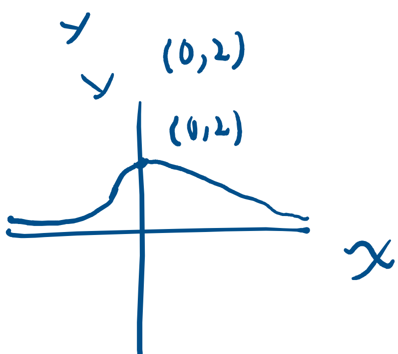
$$r(0) = \frac{2}{0^2+1} = \boxed{2}$$

$$r''(0) = \frac{16(0^2)}{(0^2+1)^3} - \frac{4}{(0^2+1)^2}$$

$$= \frac{0}{1} - \frac{4}{1}$$

$$= \boxed{-4} < 0$$

$\therefore$ $\boxed{\text{local max at } (0,2)}$



concavity

$$\boxed{r''(t) = \frac{16t^2}{(t^2+1)^3} - \frac{4}{(t^2+1)^2}}$$

concave down

$$\frac{16t^2}{(t^2+1)^3} - \frac{4}{(t^2+1)^2} < 0$$

$$\frac{4t^2}{(t^2+1)^3} - \frac{1}{(t^2+1)^2} < 0$$

$t^2+1>0$ all t $\Rightarrow 4t^2 - (t^2+1) < 0$

$t^2+1>0$ all t $\Rightarrow 4t^2-(t^2+1)<0$

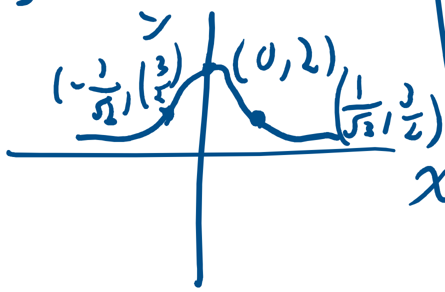
$\Rightarrow 3t^2-1<0$

$3t^2<1$

$t^2<\frac{1}{3}$

$\boxed{|t|<\frac{1}{\sqrt{3}}}$

$-\frac{1}{\sqrt{3}}<t<\frac{1}{\sqrt{3}}$

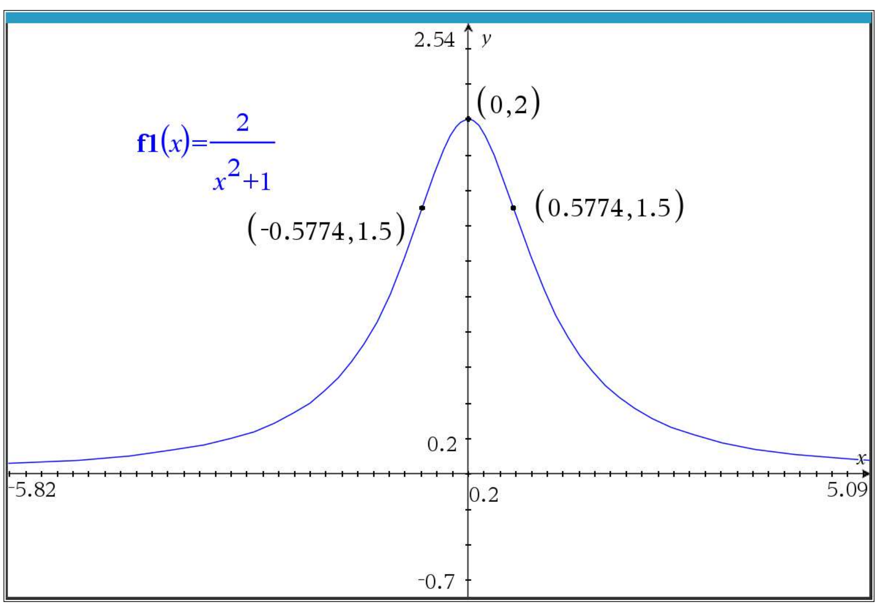


$r\left(\frac{1}{\sqrt{3}}\right)$

$=\frac{2}{\frac{1}{3}+1}=\frac{2}{\frac{4}{3}}$

$=(2)\left(\frac{3}{4}\right)$

$=\boxed{\frac{3}{2}}$



1/sqrt(3)=0.577350269189626

2.9

Memorize the procedures

Max-Min Story Problem Technique:

(a) Translate the English statement of the problem line by line into a picture (if that applies) and into math. This is often the hardest step!

(b) Identify the objective function. Look for words indicating a largest or smallest value.

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(b1) If you seem to have two or more variables, find the constraint equation. Think about the English meaning of the word "constraint," and remember that the constraint equation will have an equals sign.

(b2) Solve the constraint equation for one variable and substitute into the objective function. Now you have an equation of one variable.

(c) Use calculus to find the optimum values. (Take derivative, find critical points, test. Don't forget to check the endpoints!)

(d) Look back at the question to make sure you answered what was asked. Translate your number answer back into English.

Supplied

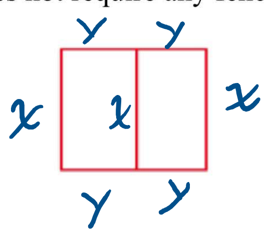
Profit has critical points when Marginal Revenue and Marginal Cost are equal.

Supplied

Average Cost has critical points when Average Cost and Marginal Cost are equal.

2.9 Exercises

1. (a) You have 200 feet of fencing available to construct a rectangular pen with a fence divider down the middle (see below). What dimensions of the pen enclose the largest total area?
   (b) If you need 2 dividers, what dimensions of the pen enclose the largest area?
   (c) What are the dimensions in parts (a) and (b) if one edge of the pen borders on a river and does not require any fencing?



Let x and y be the fence segments in the figure

$$\boxed{3x + 4y = 200 \text{ ft}}$$

$A(x,y)$ = area of rectangle

$$\boxed{A(x,y) = x(2y) = 2xy}$$

$$4y = 200 - 3x$$

$$\boxed{y = 50 - \frac{3x}{4}}$$

$$A(x) = x\left(2\left(50 - \frac{3x}{4}\right)\right)$$

$$A(x) = 2x\left(50 - \frac{3x}{4}\right)$$

$$\boxed{A(x) = 100x - \frac{3x^2}{2}}$$

Maximize A(x)

$$A'(x) = 100 - 2\left(\frac{3}{2}\right)x$$

$$\boxed{A'(x) = 100 - 3x}$$ defined for all x

$$A'(x) = 0$$

$$100 - 3x = 0$$

$$3x = 100$$

$$\boxed{x = \frac{100}{3}}$$

$$A''(x) = -3 < 0$$

$\therefore$ local max

No other critical point.

$\therefore$ global max

$$y = 50 - 3\left(\frac{100}{\ldots}\right)$$

$$y = \frac{50 - 3\left(\frac{100}{3}\right)}{4}$$

$$y = 50 - \frac{100}{4}$$

$$y = 50 - 25$$

$$\boxed{y = 25}$$

The dimension are x and $2y$, that is $\boxed{\frac{100}{3} \text{ ft and } 50 \text{ ft} \quad 33\tfrac{1}{3} \text{ ft}}$

$$3x + 4y = 200$$

$$x \geqslant 0, \; y \geqslant 0$$

$$y = 0 \Rightarrow \quad 3x = 200$$

$$x = \frac{200}{3}$$

$$0 \leq x \leq \frac{200}{3}$$

$A(0) = 0$ global min

$$A\left(\frac{100}{3}\right) = 100\left(\frac{100}{3}\right) - \frac{3\left(\frac{100}{3}\right)^2}{2} = \frac{100^2}{3} - \frac{3}{2}\left(\frac{100}{3}\right)^2$$

$$= \frac{100^2}{3}\left(1 - \frac{1}{2}\right) = \frac{100^2}{6}$$

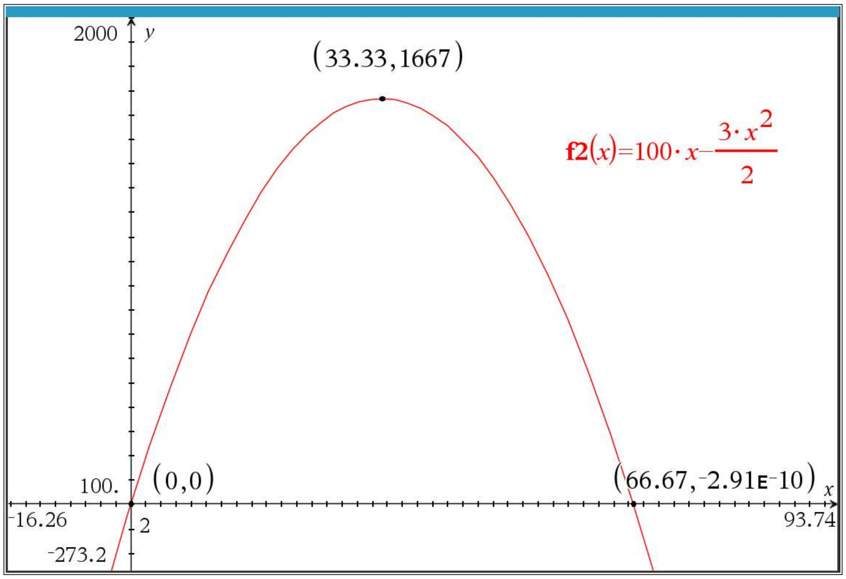
$$\approx \boxed{1,666.67 \text{ ft}^2}$$

(100^2)/6=
1,666.6667

$A\left(\frac{200}{3}\right) = 0$ global min

$A\left(\frac{200}{3}\right) = 0$ global min

Here is the graphical estimation of the maximum area and the corresponding value of x.



Without calculus, we could have solved this problem by finding the exact coordinates of the vertex of the parabola.